December 20th 1796

497 **TRADE AND ECONOMICS.** HAWNEY (William) The Complete Measurer: Very useful for all TRADESMEN; especially CARPENTERS, BRICKLAYERS, PLAISTERERS, PAINTERS, GLASIERS, MASONS, &c. *Printed for J. F. and C. Rivington, &c.*, 1789. £3 3s

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THE

COMPLETE MEASURER:

John OR, THE Hall 179

WHOLE ART OF MEASURING.

IN TWO PARTS.

The Firſt PART teaching

DECIMAL ARITHMETICK, with the *Extraction* of the SQUARE and CUBE ROOTS:

And alſo the *Multiplication of Feet and Inches*, commonly called CROSS MULTIPLICATION.

The Second PART teaching to

Meaſure all Sorts of SUPERFICIES and SOLIDS, by *Decimals*; by *Croſs Multiplication*, and by *Scale and Compaſſes*: Alſo the Works of ſeveral Artificers, relating to *Building*; and the Meaſuring of *Board* and *Timber*. Shewing the common Errors.

And ſome *Practical* QUESTIONS.

The *Sixteenth* EDITION reviſed and corrected.

To which is added,

An APPENDIX. 1. *Of* Gauging. 2. *Of* Land-Meaſuring.

Very uſeful for all Tradeſmen; eſpecially Carpenters, Bricklayers, Plaſterers, Painters, Joiners, Glaziers, Maſons, *&c.*

By WILLIAM HAWNEY, *Philomath.*

Recommended by the Rev. Dr. John Harris, *F. R. S.*

LONDON:

Printed for J. F. and C. RIVINGTON, T. LONGMAN, B. LAW, S. BLADON, G. G. J. and J. ROBINSON, J. JOHNSON, W. GOLDSMITH, R. BALDWIN, J. and J. TAYLOR, E. NEWBERY, SCATCHERD and WHITAKER, and G. and T. WILKIE. 1789.

I Have perused this BOOK, and recommend it to the Publick as a very useful One.

J. Harris, *D.D.*

THE

PREFACE.

HAVING perufed feveral Books concerning the Menfuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book fo perfect, as to give any tolerable Satisfaction to a Learner; and I having practifed and taught Meafuring for feveral Years, and thereby gained Experience and Knowledge in that Art, having learned fome Things from one Author, and fome Things from another, I began to think of digefting my Thoughts into fome fuch Method as might give a Learner full Satisfaction, without being at the Charge of buying fo many Books; and being importuned thereunto by fome Friends, I fell to work, and at laft brought them to that Perfection you here find in the following Work.

1. As to the *Decimal Arithmetick*, I have been as concife as the Matter would well bear, to make it plain.

2. As to the Multiplying of Feet and Inches, commonly called *Cross Multiplication*, my Method differs from that which is usually taught in other Authors, as being (I think) much shorter and plainer.

3. In measuring of Superficies and Solids, I have given the Demonstration of the Rules, which I thought might be very acceptable to the Ingenious; for, indeed, I always look upon the Writing of a Rule without a Demonstration (in any Part of the Mathematicks) to be but lame and defective; for want of knowing the Reason of the Rule, a Learner may commit great Errors; besides, when a Learner knows the Reason of the Rules, he may retain them better in his Memory. The Rule for measuring a Prismoid and Cylindroid, I had out of Mr. *Everard*'s *Art of Gauging*; but the Reason he does not shew, neither have I found it in any other Author; but that the Method is true, I have endeavoured to make plain.

The Demonstration of the Rules for finding the Area of an Ellipsis and Parabola; also the Demonstration of the Rules for finding the solid Content of the Frustum of a Cone and Pyramid, the Solidity of a Globe of a Spheroid, a Parabolic Conoid, and of a Parabolic Spindle, and their Frustums, I had from the ingenious Mr. *Ward*'s *Young Mathematician's Guide*; where the curious and ingenious Reader may see many other Demonstrations algebraically performed. I have also demonstrated the Rule for finding the Solidity of a Globe, out of *Pardie*'s *Elements of Geometry* (Book the 5th, Art. the 33d) published in *English* with many Additions,

Additions, by the Reverend Dr. *Harris*, F. R. S. and the ſame is alſo done out of *Sturmius*'s *Matheſis Enucleata*; ſo that the ingenious Reader may uſe which of thoſe Ways he likes beſt.

The Scale ſuppoſed to be uſed in all the Operations, is the Line of Numbers, commonly called *Gunter*'s *Line*, which is upon the ordinary Two-Feet or Eighteen-Inch Rules, commonly uſed by the Carpenters, Maſons, &c. becauſe I thought it needleſs, as well as impertinent, to write the Uſe of Sliding-Rules, or any other particular Scales, they being ſufficiently treated of by ſeveral Authors; *viz.* by the above-named Mr. *Everard*, in his *Art of Gauging* above-mentioned, where you have the Uſe of a Sliding-Rule in Arithmetick, Geometry, in Meaſuring of Superficies and Solids, Gauging, &c. Likewiſe Mr. *Hunt* has written largely of the Uſes of his Sliding Rule, in Arithmetick, Geometry, Trigonometry, Gauging, Dialling, &c. There are ſeveral others who have explained the Uſe of their own Rules; ſo that the more curious Reader may find full Satisfaction in thoſe Authors.

One Thing I have omitted in the Book, which I think may not be very improperly inſerted in this Place; that is, how to find a Number upon the Line. If the Number you would find conſiſts only of Units, then the Figures upon the Line repreſent the Number ſought: Thus, if the Number be 1, 2, 3, &c. then 1, 2, 3, &c. upon the Line, repreſents the Number ſought. But if the Number conſiſts of two Figures, that is, of Units and Tens, then the Figure upon the Rule ſtands for

the Tens, and the large Divisions stand for the Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Divisions (counted forwards) is the Point representing 34; and if 340 were to be found, it will be at the same Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and four of the smaller Divisions (counted forward) is the Point representing 304. If the Number consists of four Places, or Thousands, then the Figure upon the Line stands for Thousands, and the larger Divisions are Hundreds, the lesser Divisions are Tens, and the tenth Parts of those lesser Divisions are Units. Thus, if 2735 were to be found, then the 2 to 2000; and the 7 larger Divisions (counted forward) is 700 more; and 3 of the lesser Divisions is 30 more; and half of one of the lesser Divisions is 5 more, which is the Point representing 2735. You must remember, that between each Figure upon the Line there are 10 Parts, which I call the larger Divisions; and each of those larger Divisions are subdivided (or supposed so to be) into 10 other Parts, which I call the smaller Divisions; and each of those Parts supposed so to be subdivided again into 10 other Parts, *&c.* You must also remember, that if 1 in the Middle of the Line stands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End signifies 1, then 1 in the Middle stands for 10, and 1 at the upper End is 100, *&c.*

There is one Thing more which I would have my Reader to understand; and that is, how to find all

all ſuch proportional Numbers made uſe of in the Proportions about a Circle, and of a Cylinder, and in other Places; which Thing may be of good Uſe to know how to correct a Number, which may happen to be falſe printed, or to enlarge any Number to more decimal Places, for more Exactneſs; for though I have mentioned what ſuch Numbers are, yet I have not ſhewn how to find them, which a Learner may be a little at a Nonplus to do; though they are eaſily found by the Rules there laid down. I ſhall therefore give two or three Examples, in this Place, of finding ſuch Numbers, which may enable my Reader to find out the reſt.

And, firſt, let it be required to find the Area of a Circle, whoſe Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.14159265, *&c.* of which 3.1416 is ſufficient in moſt Caſes. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, multiply 1.5708 by .5 (*viz.* half 3.1416 by half 1) and the Product is .7854, which is the Area of the Circle, whoſe Diameter is 1.

Again; if the Area be required when the Circumference is 1, firſt find what the Diameter will be, thus: 3.1416 : to 1 : : ſo is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is .159154 by .5, and the Product is .079577, which is the Area of a Circle whoſe Circumference is 1.

If

If the Area be given, to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7854 is .8862 which is the Side of a Square equal when the Diameter is 1. And if you extract the Square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle whose Circumference is 1.

If the Side of a Square within a Circle be required, if you square the Semidiameter, and double that Square, and out of that Sum extract the Square Root, that shall be the Side of the Square which may be inscribed in that Circle; so if the Diameter of the Circle be 1, then the half is .5; which squared, is .25; and this, doubled, is .5, whose Square Root is .7071, the Side of the Square inscribed.

Again; If the Diameter of a Globe be 1. to find the Solidity. In Sect. XI. Chap. II. it is demonstrated, that the Globe is $\frac{2}{3}$ of a Cylinder of the same Diameter and Altitude: Thus, if the Cylinder's Diameter be 1, and its Altitude or Length be also 1, find the Solidity thereof, and take $\frac{2}{3}$ of it, and that will be the Solidity of the Globe required. Now if the Diameter be 1, the Area of the Circle, or Base of the Cylinder, is .7854 (as is above shewn) which multiplied by 1, the Altitude of the Cylinder, and the Product is also .7854, the Solidity of the Cylinder; $\frac{2}{3}$ whereof is .5236, which is the Solidity of the Globe, whose Diameter is 1.

From

From what has been ſaid, the Reader may eaſily perceive how all other proportional Numbers are found, and may examine them at his Pleaſure.

I ſhall not enlarge any farther upon the Matter, but leave the Book to ſpeak for itſelf; and if it prove beneficial to the ingenious Practitioners, I have my Deſire. So, wiſhing my ingenious Reader good Succeſs in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Deſire of his Well-wiſher,

W. HAWNEY.

CON-

CONTENTS.

PART I.

PART II. CHAP. I.

CHAP.

CONTENTS.

CHAP. II.

Of Solid Measure.

CHAP. III.

The Measuring of Works relating to Building.

CHAP. IV.

4. *Of*

CONTENTS.

CHAP. V.

APPENDIX.

ERRATA.

Page 23, line 1, for *fifth*, read *fourth*. p. 78, in the fig. for 78, r. 7.8. p. 85, in the fig. for 66, r. 6.6. p. 87, for 12.2 in the fig. r. 12.64. p. 122, in the fig. for 189, r. 18.9, and for 35, r. 3.5. p. 203, in the fig. for *a*, r. *d*.

THE

THE

The Complete MEASURER.

PART I.

CHAP. I.

Notation of DECIMALS.

A DECIMAL Fraction is an artificial Way of ſetting down and expreſſing Natural, or Vulgar fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annexed to it, and muſt therefore be either 10, 100, 1000, 10000, *&c.* and conſequently in writing down a Decimal Fraction, there is no Neceſſity for writing down the Denominator; as by bare Inſpection, it is certainly known, conſiſting of an Unit with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus .25, its Denominator being known to be an Unit with two Cyphers; becauſe there are two Figures in the Numerator. In like Manner, $\frac{125}{1000}$ may be thus written, .125; and $\frac{3575}{10000}$ thus, .3575; and $\frac{75}{1000}$ thus, .075; and $\frac{65}{10000}$ thus, .0065.

As whole Numbers increaſe in a decuple or ten-fold Proportion, towards the Left-hand, ſo, on the contrary, Decimals decreaſe towards the Right-hand in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thouſandth Parts.	Ten Thouſandth Parts.	Hundred Thouſandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0,	1	2	3	4	5	6

Hence it appears, that Cyphers put on the Right-hand of whole Numbers, increaſe the Value of thoſe Numbers in a decuple (or ten-fold) Proportion: But being annexed to the Right-hand of a Decimal Fraction, neither increaſe nor decreaſe the Value of it: So $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$ or .25. And, on the contrary, though in whole Numbers, Cyphers before them, neither increaſe nor diminiſh the Value; yet Cyphers before a decimal Fraction diminiſh its Value in a decuple Proportion: For .25, if you put a Cypher before it, becomes $\frac{025}{1000}$ or .025: And .125 is $\frac{00125}{100000}$, by prefixing two Cyphers thus, 00125. And therefore, when you are to write a decimal Fraction, whoſe Denominator hath more Cyphers than there are Figures in the Numerator,

the Places of ſuch Figures muſt be ſupplied by placing Cyphers before the Figures of your Numerator; as, ſuppoſe $\frac{19}{1000}$ were to be written down, without its Denominator; here, becauſe there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore put a Cypher before 19, and ſet it down thus, .019.

The integers are ſeparated from the Decimals ſeveral Ways, according to Men's Fancies; but the beſt and moſt uſual Way is a Point or Period; and if there be no whole Number, then a Point before the Fraction is ſufficient: Thus, if you were to write down 317 $\frac{217}{1000}$ it may be thus expreſſed, 317.217; and 59 $\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus, .0075, &c.

CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Caſes: 1ſt, To reduce a Vulgar Fraction to a Decimal. 2dly, To find the Value of a Decimal in the known Parts of Coin, Weights, Meaſures, &c. 3dly, To reduce Coin, Weights, Meaſures, &c. to a Decimal. Of theſe in their Order.

I. *To reduce a Vulgar Fraction to a Decimal.*

The RULE.

As the Denominator of the given Fraction is to its Numerator, ſo is an Unit (with a competent Number of Cyphers annexed) to the Decimal required.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Re-

sult by the Denominator, the Quotient is the Decimal equivalent to the Vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduced to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal required, and is equivalent to $\frac{3}{4}$ given.

Note, That so many Cyphers as you annex to the given Numerator, so many Places must be pricked off in the Decimal found; and if it should happen, that there are not so many Places of Figures in the Quotient, the Deficiency must be supplied, by prefixing Cyphers to the Quotient Figures, as in the next Example.

Example 2. Let $\frac{3}{573}$ be reduced to a Decimal having six Places.

To the Numerator annex six Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was required to have six Places, therefore you must put two Cyphers before it, and then it will be 005235, which is the Decimal required, and is equivalent to $\frac{3}{573}$.

See the Work of these two Examples.

```
4)3·00(·75        573)3·000000(005235
  28                      . . .
  ——              ————————————
  20                 1350
  20                   2040
  ——                     3210
  . .                ————————
                          345
```

In the second Example there remains 345, which Remainder is very insignificant, it being less than $\frac{1}{100000}$ Part of an Unit, and therefore is rejected.

II. *To*

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Measure, &c.*

The RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off ſo many Places to the Right-hand as there were Places in the Decimal given; and multiply thoſe Figures pricked off by the Number of Parts in the next inferior Denomination, and prick off ſo many Places as before, and ſo continue to do, till you have brought it to the loweſt Denomination required.

Example 1. Let .7565 of a Pound Sterling be given to be reduced to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and by 4, as the Rule directs, and always prick off four Places to the Right-hand, and you will find it make 15 *s.* 1 *d.* 2 *q.* See the Work.

$$\begin{array}{rr}
 & .7565 \\
 & 20 \\
\hline
s. & 15.1300 \\
 & 12 \\
\hline
d. & 1.5600 \\
 & 4 \\
\hline
q. & 2.2400
\end{array}$$

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the firſt Figure, (or Place of Primes) and it makes ſo many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5, for the 5 add another Shilling to the former Shillings; then

for every Unit in the ſecond Place count ten, and to that add the Figure in the third Place, and reckon that ſo many Farthings; but if they make above 13, abate 1; and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings, Pence, and Farthings.

Firſt, Double your 6, and it makes it 12*s.* then take 5 out of 9, and for that reckon another Shilling, and it makes 13*s.* and the 4 remaining is four Tens, and the 5 makes 45, which being above 38, you muſt therefore caſt away 2, and there reſt 43 Farthings, which is 10*d.* $\frac{3}{4}$. So the Anſwer is 13*s.* 10*d.* $\frac{3}{4}$

	l.		*s.*	*d.*
So the Value of	.725	=	14	6
And the Value of	.878	=	17	$6\frac{3}{4}$
And the Value of	.417	=	8	4

And ſo of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Penny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the Right-hand, and you will find the Anſwer to be 7 *oz.* 3 *pwts.* 10 *gr. fere.* See the Work.

```
 .59755
     12
 -------
 7.17060
      20            oz. pwts. gr.
 ------- Facit      7    3    9.888.
 3.41200
      24
 -------
  164800
  82400
 -------
 9.88800
```

Let

Let .43569 of a Ton be reduced to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Anſwer will be 8 *C.* 2 *qrs.* 23 *lb. ferè.*

```
 .43569
     20
 ------
8.71380
      4          C.  qrs.  lb.
 ------   Facit  8    2    23.9456
2.85520
     28
 ------
23.94560
```

Let .9595 of a Foot be reduced into Inches and Quarters.

```
 .9595
    12
 -----
11.5140
      4  Facit 11 Inches, 2 Quarters.
 -----
2.0560
```

III. *To reduce the known Parts of Money, Weight, Meaſure,* &c. *to a Decimal.*

The RULE.

To the Number of Parts of the leſſer Denomination given, annex a competent Number of Cyphers, and divide by the Number of ſuch Parts that are contained in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal ſought.

Example

Example 1. Let 6 *d.* be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers (suppose 3), and divide the Result by 240 (the Pence in a Pound), and the Quotient is the Decimal required.

240)6.00|0(.025

1200

Facit .025

Example 2. Let 3 *d.* $\frac{3}{4}$ be reduced to the Decimal of a Pound, having six Places.

In 3 *d.* $\frac{3}{4}$ there are fifteen Farthings, therefore to 15 annex six Cyphers (because there are to be six Places in the Decimal required), and divide by 960 (the Farthings in a Pound), and the Quotient is .015625.

96|0)15.00000|0(.015625

540
600
240
480

Example 2. Let 3 $\frac{1}{4}$ Inches be reduced to the Decimal of a Foot, consisting of four Places.

In 3 $\frac{1}{4}$ Inches, there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48 (the Quarters in a Foot), and the Quotient is .2708.

48)13.0000(2708

340
400

16

Example

Example 4. Let 9 *C.* 1 *qr.* 16 *lb.* be reduced to the Decimal of a Ton, having six Places.

C.	*qu.*	*lb.*	
9	1	16	2240)1052.00000|0(469642
4			. . . . . .
37 *qrs.*			15600
28			21600
			14400
302			96000
75			6400
1052 Pounds.			1920

Facit .469642.

CHAP. III.

Addition of DECIMALS.

ADDITION of Decimals is performed the same Way as Addition of whole Numbers, only you must observe to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, *&c.*

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

```
    317.25
     17.125
    275.5
     47.3579
     12.75
    --------
Sum 669.9829
```

This is so plain, that I think more Examples needless.

CHAP.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is likewiſe performed the ſame Way as in whole Numbers, reſpect being had (as in Addition) to the right placing the Numbers, as in the following Examples.

	(1)		(2)
From	212.0137	From	201.1250
Subtr.	31.1275	Subtr.	5.5785
Reſts	180.8862	Reſts	195.5465
Proof	212.0137	Proof	201.1250

	(3)		(4)
From	2051.315	From	30.5
Subtr.	79.172	Subtr.	7.2597
Reſts	1972.143	Reſts	23.2403
Proof	2051.315	Proof	30.5

Note, If the Number of Places in the Decimals be more in that which is to be ſubtracted, than in that which you ſubtract from, you muſt ſuppoſe Cyphers to make up the Number of Places, as in the fourth Example.

CHAP.

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is alſo performed the ſame Way as Multiplication of whole Numbers; but to know the Value of the Product, obſerve this Rule.

Cut off, or ſeparate by a Comma or Point, ſo many decimal Places in the Product, as there are Places of Decimals in both Factors, *viz.* both in the Multiplicand and Multiplier; which I ſhall further explain in the following Examples.

Let 3.125 be multiplied by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8,59375: And becauſe there were three Places of Decimals pricked off in the Multiplicand, and two Places in the Multiplier, therefore you muſt prick off five Places of Decimals in the Product, as you may ſee by the Work.

```
  3.125
   2.75
  -----
  15625
 21875
 6250
-------
8.59375
```

Let

Let 79.25 be multiplied by .459.

In this Example, becauſe two Places of Decimals are pricked off in the Multiplicand, and three in the Multiplier, therefore there muſt be five pricked off in the Product.

```
 79.25
  .459
------
 71325
39625
31700
------
36.37575
```

Let .135272 be multiplied by .00425.

In this Example, becauſe in the Multiplicand are ſix decimal Places, and in the Multiplier five Places; therefore in the Product there muſt be eleven Places of Decimals; but when the Multiplication is finiſhed, the Product is but 57490600, *viz.* only eight Places; therefore, in this Caſe, you muſt put three Cyphers before the Product Figures, to make up the Number of eleven Places: So the true Product will be .00057490600.

```
   .135272
    .00425
----------
    676360
   270544
  541088
----------
.00057490600
```

More

More Examples for Practice.

.001472	.017532
.1045	347
7360	122724
5888	70128
14720	52596
.0001538240	6.083604
279.25	32.0752
.445	.0325
139625	1603760
111700	641504
111700	962256
124.26625	1.04244400
4.443	20.0291
15.98	35.45
35544	1001455
39987	801164
22215	1001455
4443	600873
70.99914	710.031595
7.3564	.75432
.0126	.0356
441384	452592
147128	377160
73564	226296
.09269064	.026853792

Contracted Multiplication of Decimals.

Becauſe in Multiplication of Decimal Parts, and mixed Numbers, there is no need to expreſs all the Figures of the Product, but in moſt Caſes two, three, or four Places of Decimals will be ſufficient; therefore, to contract the Work, obſerve the following

RULE.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, which you intend to keep in the Product; then invert the Order of all the other Figures; that is, write them all the contrary Way; and, in multiplying, begin always at that Figure in the Multiplicand which ſtands over the Figure you are then multiplying withal, and ſet down the firſt Figure of each particular Product directly one under the other: But yet a due Regard muſt be had to the Increaſe ariſing from the Figures on the Right-hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four Places retained in the Decimals of the Product.

Firſt, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the reſt of the Figures quite contrary to the uſual Way, as in the following Work: Then begin to multiply, firſt the 5 which is left out (only with regard to the Increaſe which muſt be carried from it); ſaying, 8 times 5 is 40; carry 4 in your Mind, and ſay, 8 times 4 is 32, and 4 I carry, is 36; ſet down 6, and carry 3, and proceed through the reſt of the Figures as in common Multiplication:

tiplication: Then begin to multiply with 2; ſaying, 2 times 4 is 8, for which I carry 1, (becauſe it is above 5), and ſay, 2 times 6 is 12, and 1 that I carry is 13; ſet down 3, and carry 1, and proceed through the reſt of the Figures: Then multiply with 1; ſaying, once 6 is 6, for which I carry 1, and ſay, once 8 is 8, and 1 is 9; ſet down 9, and proceed: Then multiply with 7; ſaying, 7 times 8 is 56, for which carry 6 (becauſe it is above 55), and ſay, 7 times 3 is 21, and 6 that I carry is 27; ſet down 7, and carry 2, and proceed; then multiply with 5; ſaying, 5 times 3 is 15, for which carry 2, and ſay, 5 times 2 is 10, and 2 I carry is 12, which ſet down, and add all the Products together; and the total Product will be 19.6107. See the Work.

$$
\begin{array}{r}
2.38645 \\
5712.8 \\
\hline
19.0916 \\
4773 \\
239 \\
167 \\
12 \\
\hline
19.6107
\end{array}
$$

Note, That in multiplying the Figure left out every Time next the Right-hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards 30, carry 3, &c.

I have here ſet down the Work of the laſt Example, wrought by the common Way, by which you may ſee both the Reaſon and Excellency of this Way, all the Figures on the Right-hand of the Line being wholly omitted.

$$\begin{array}{r@{}l}
\multicolumn{2}{r}{2.38645} \\
\multicolumn{2}{r}{8.2175} \\
\hline
11 & |93225 \\
167 & |0515 \\
238 & |645 \\
4772 & |90 \\
190916 & |0 \\
\hline
19.6106 & |52875
\end{array}$$

Example 2. Let 375.13758 be multiplied by 16.7324, ſo that the Product may have but four Places of Decimals.

Firſt, ſet 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand (becauſe four Places of Decimals were to be preſerved), and write all the reſt of the Figures backward. Then multiply all the Figures of the Multiplicand by 1, after the common Way. Then begin with the ſecond Figure of the Multiplier 6; ſaying, 6 times 8 is 48, for which I carry 5 (in reſpect of the 8 left out), and 6 times 5 is 30, and 5 that I carry is 35; ſet down 5 and carry 3, and proceed after the common Method. Then begin with 7, the third Figure of the Multiplier, and ſay 7 times 5 is 35, for which carry 4, and ſay 7 times 7 is 49, and 4 I carry is 53; ſet down 3 under the firſt, and carry 5, and proceed as before. Then begin with 3, the fourth Figure of the Multiplier, and ſay 3 times 7 is 21, carry 2, and ſay 3 times 3 is 9, and 2 I carry is 11; ſet down 1 and carry 1, and proceed as before. Then begin with 2, the fifth Figure, and ſay 2 times 3 is 6, for which I carry 1, and ſay 2 times 1 is 2, and 1 I carry is 3; ſet down 3, and 2 times 5 is 10; ſet down 0, and carry 1, and proceed as before. Then begin with 4, the laſt Figure of the Multiplier, and ſay 4 times 1 is 4, for which I carry nothing, becauſe it is leſs than 5: Then

Then ſay 4 times 5 is 20; ſet down 0, and carry 2, and proceed thro' the reſt of the Figures of the Multiplicand. Then add all up together, and the Product is 6276.9520. See the Work.

375·13758 the Multiplicand.
4237.61 the Multiplier reverſed.

37513758 the Product with 1.
22508255 the Product with 6 increaſed with 6 × 8.
2625963 the Product with 7 increaſed with 7 × 5.
112541 the Product with 3 increaſed with 3 × 7.
7503 the Product with 2 increaſed with 2 × 3.
1500 the Product with 4 increaſed with 0.

6276.9520 the Product required.

Let the ſame Example be repeated, and let only one Place in Decimals be pricked off.

375·13758 the Multiplicand.
4237.61 the Multiplier reverſed.

37514 the Product by 1 with the Increaſe of 1 × 7.
22508 the Product with 6 increaſed with 6 × 3.
2626 the Product with 7 increaſed with 7 × 1.
113 the Product with 3 increaſed with 3 × 5.
7 the Product with 2 increaſed with 2 × 7.
1 the Increaſe only of 4 × 3.

6276.9 the Product is the ſame as before.

More Examples for Practice.

Multiply 395.3756 by .75642; and prick off four Places in Decimals.

395.3756	the Multiplicand.
24657.	the Multiplier reversed.
2767629	the Product by 7 increased with 7 × 6.
197688	the Product by 5 increased with 5 × 5.
23722	the Product by 6 increased with 6 × 7.
1581	the Product by 4 increased with 4 × 3.
79	the Product by 2 increased with 2 × 5.
299.0699	the Product required.

Let the same Example be repeated, and let there be only one Place of Decimals.

395.3756	
24657.	
2767	the Product by 7 increased with 7 × 3.
198	the Product by 5 increased with 5 × 5.
24	the Product by 6 increased with 6 × 9 + 6 × 5.
2	the Increase of 4 × 9 + 4 × 3.
299.1	the Product.

Characters,

Characters, and their Signification.

Note, That this Mark + ſignifies Addition; as 8 + 5, that is, 8 more 5, or 8 added to 5; and 8 + 3 + 7, denotes theſe Numbers are to be added into one Sum.

This Mark — ſignifies Subtraction, as 9 — 4 ſignifies that 4 is to be taken from 9.

This Mark × ſignifies Multiplication, as 7 × 5 ſignifies that 7 is to be multiplied by 5.

This Mark ÷ ſignifies Diviſion, as 12 ÷ 4 ſignifies 12 is to be divided by 4.

This Mark = ſignifies Equality, or Equation; that is, when = is placed between Numbers, or Quantities, it denotes them to be equal, as 7 + 5 = 12, that is, 7 more 5 is equal to 12; and 15 — 7 = 8, that is, 15 leſs by 7, is equal to 8, or ſubtract 7 from 15 and there remains 8.

This Mark :: is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two middle Terms or Numbers in Proportion; thus 4 : 20 :: 6 : 30, to be thus read, as 4 is to 20, ſo is 6 to 30.

CHAP. VI.

Diviſion of DECIMALS.

DIVISION of Decimals is performed after the ſame Manner as Diviſion of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty; for the reſolving of which, obſerve either of the following

RULES.

RULES.

I. The firſt Figure in the Quotient muſt be of the ſame Denomination with that Figure in the Dividend which ſtands (or is ſuppoſed to ſtand) over the Unit's Place in the Diviſor, at the firſt ſeeking.

II. When the Work of Diviſion is ended, count how many Places of Decimal Parts there are in the Dividend more than in the Diviſor; for that Exceſs is the Number of Places which muſt be ſeparated in the Queſtion for Decimals. But if there be not ſo many Figures in the Quotient as there are in the ſaid Exceſs, that Deficiency muſt be ſupplied, by placing Cyphers before the ſignificant Figures, towards the Left-hand, with a Point before them; and thus you will plainly diſcover the Value of the Quotient.

Theſe following Directions ought alſo to be carefully obſerved.

If the Diviſor conſiſts of more Places than the Dividend, there muſt be a competent Number of Cyphers annexed to the Dividend, to make it conſiſt of as many (at leaſt) or more Places of Decimals than the Diviſor; for the Cyphers added muſt be reckoned as Decimals.

Conſider whether there be as many Decimal Parts in the Dividend as there are in the Diviſor; if there be not, make them ſo many, or more, by annexing of Cyphers.

In dividing of whole or mixed Numbers, if there be a Remainder, you may bring down more Cyphers; and, by continuing your Diviſion, carry the Quotient to as many Places of Decimals as you pleaſe.

Theſe Things being conſidered, I ſhall proceed to the Practice of Diviſion of Decimals, which I ſhall endeavour to explain in as familiar and eaſy a Method as poſſible.

Example

Example 1. Let 48 be divided by 144.

In this Example the Divisor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers (*viz.* four), with a Point before them, and divide in the usual Way.

```
144)48.0000(.3333
    -------
     480
      480
       480
    -------
        48
```

But, first, in seeking how often 144 in 48.0 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the first Place of Decimals; therefore the first Figure in the Quotient is in the first Place of Decimals: Or, by the second Rule, there being four Places of Decimals in the Dividend, and none in the Divisor; so the Excess of decimal Places in the Dividend, above that in the Divisor, is four; so that when the Division is ended, there must be four Places of Decimals in the Quotient. See the Work.

Example 2. Let 217.75 be divided by 65.

First, in seeking how often 65 in 217 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the Unit's Place of the Dividend; therefore the first Figure in the Quotient will be Units, and all the rest Decimals: Or, by the second Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Divisor, therefore the Excess of Decimal Places in the Dividend, above the Divisor, is two; so when the Division is ended, separate two Places in the Quotient, towards the Right-hand by a Point. See the Work.

65)217·75(3·35
   · · ·

227
 325

· · ·

Example 3. Let 267.15975 be divided by 13.25.

13·25)267.15975(20.163
   · · · · ·

2159
 8347
  3975

· · · ·

In this third Example, the Unit's Place of the Divisor, falls under 6, the Ten's Place of the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is Tens: Or, by the ſecond Rule, the Exceſs of Decimal Places in the Dividend, above the Diviſor, is three; there being five Places of Decimals in the Dividend, and but two in the Diviſor, ſo there muſt be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89.

375·89)15.675159(.0417
   · ·

63955
263669

  546

In

In this fifth Example, the Unit's Place of the Divisor, falls under 7, the second Place of Decimals in the Dividend; therefore (by the first Rule) the first Figure in the Quotient is in the second Place of Decimals; so that you must put a Cypher before the first Figure in the Quotient; and by the second Rule, the Excess of decimal Places in the Dividend above the Number of decimal Places in the Divisor is 4; for the decimal Places in the Dividend are 6, and the Number of Places in the Divisor but two; therefore there must be four Places of Decimals in the Quotient: But the Division being finished after the common Way, the Figures in the Quotient are but three, therefore you must put a Cypher before the significant Figures.

Example 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
          . . .
       ---------
        4806
         7654
          9190
           11080
       ---------
             304
```

In this Example, the Divisor being a Decimal, the 1st Figure falls under the Ten's Place in the Dividend, therefore the Units (if there had been any) should fall under the Hundred's Place in the Dividend, and so the first Figure in the Quotient is Hundreds. And, by the second Rule, there being four Places of Decimals in the Dividend, and as many in the Divisor, so the Excess is nothing; but in dividing I put two Cyphers to the Remainders, and continue the Division to two Places further; so I have two Places of Decimals. See the Work.

Example

Example 6. Let .125 be divided by .0457.

```
.0457)·1250000(2.735
       914···
       ------
       3360
       3199
       ------
        1610
        1371
        ------
         2390
         2285
         ------
          105
```

In this Example, the Unit's Place of the Divisor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the first Figure of the Quotient is Units. And, by the second Rule, there being seven Places of Decimals in the Dividend, and but four Places in the Divisor, so the Excess is three; therefore there must be three Places of Decimals in the Quotient.

I shall set down only the Work of some few Examples more, and so proceed to *Contracted Division.*

```
.00456).0000059791(.00131
                ··
             ------
             1419
              511
             ------
               55
```

Let

Let Unity be divided by 282.

```
282)1.0000000(.0035461 ferè.
      .....
    ---------
      1540
       1300
        1720
         280


.325).400000(1.2307
         ...
     --------
     750
     1000
      2500
     --------
       225


.042)495.00000(11785.71
         ......
     -----------
     75
     330
      360
       240
        300
         60
     -----------
         18
```

Division of DECIMALS *contracted.*

IN Division of Decimals the common Way, when the Divisor hath many Figures, and it is required to continue the Division till the value of the Remainder be but small, the Operation will sometimes be long and tedious, but may be excellently contracted by the following Method.

The RULE.

By the first Rule of this Chapter (*Page* 20), find what is the Value of the first Figure in the Quotient: then, by knowing the first Figure's Denomination, you may have as many or as few Places of Decimals as you please, by taking as many of the Left-hand Figures of the Divisor as you think convenient for the first Divisor; and then take as many Figures of the Dividend as will answer them; and, in dividing, omit one Figure of the Divisor at each following Operation. A few Examples will make it plain.

Example 1. Let 721.17562 be divided by 2.257432; and let there be three Places of Decimals in the Quotient.

```
2.25743)721.175|62(319.467
 .....  677229
        ------
         43946
         22574
        ------
         21372
         20317
        ------
          1055
           903
         -----
           152
           135
          ----
            17
            15
          ----
             2
```

In this Example, the Unit's Place of the Divisor falls under the Hundred's Place in the Dividend, and it is required, that three Places of Decimals be in the Quotient, so there must be six Places in all; that is, three Places of whole Numbers, and three Places of Decimals. Then, because I can have the Divisor in the first six Figures of the Dividend, I cut off the 62 with a Dash of the Pen, as useless; then I seek how often the Divisor is in the Dividend, and the Answer is three times; put 3 in the Quotient, and multiply and subtract as in common Division, and the Remainder is 43946. Then prick off the 3 in the Divisor, and seek how often the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and subtract, and the next Remainder is 21372. Then prick off the 4 in the Divisor, and seek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient; multiply thus, saying 9

 times

times 4 is 36, for which I carry 4 (in respect of the 4 last pricked off), and 9 times 7 is 63, and 4 is 67; set down 7, and carry 6, and so proceed till the Division be finished, always respecting the Increase made from the Figures pricked off. Observe the Work, which will better inform you than many Words.

2.25743)721.17562(319.467

677229	
43946	6
22574	3
21372	32
20316	87
1055	450
902	972
152	4780
135	4458
17	03220
15	80201
1	23019

I have set down the Work of this last Example at large, according to the common Way, that thereby the Learner may see the Reason of the Rule, all the Figures on the Right-hand Side the perpendicular Line being wholly omitted.

Example

Example 2 Let 5171.59165 be divided by 8.758615; and let it be required, that four Places of Decimals be pricked off in the Quotient.

```
8.758615)5171.5916|5(590.4577
•••••
              43793075
              --------
               7922841
               7882754
              --------
                 40087
                 35034
                 -----
                  5053
                  4379
                  ----
                   674
                   613
                   ---
                    61
                    61
                    --
                    ••
```

In this Example, I can't have 8, the firſt Figure in the Diviſor, in 5, the firſt Figure of the Dividend; ſo that the Unit's Place of the Diviſor falls under the Hundred's Place in the Dividend; ſo that there will be ſeven Figures in the Quotient; that is, three of whole Numbers, and four of Decimals; therefore there muſt be ſeven Figures in the Diviſor (becauſe the Number of Places in the Diviſor and Quotient will be equal), and there muſt be eight Places in the Dividend; ſo that I cut off the Figure 5 with a Daſh, as uſeleſs. Thus having proportioned the Dividend to the Diviſor, and both to the Number of Places or Figures deſired in the Quotient, I proceed to divide as before; ſaying how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and ſub-

tract, and the Remainder is 7922841. Then I prick off the firſt Figure in the Diviſor, 5, and ſeek how often the remaining Figures of the Diviſor in the aforeſaid Remainder, which I find nine times; put 9 in the Quotient, and multiply thereby, ſaying 9 times 5 (the Figure pricked off) is 45, for which I carry 5, and ſay 9 times 1 is 9, and 5 I carry is 14; ſet down 4, and carry 1, and proceed to multiply the reſt of the Figures, and ſubtract, and the Remainder will be 40087. Then prick off the Figure 1, and ſeek how often 87586 in the Remainder 40087, the Anſwer will be 0; ſo put 0 in the Quotient, and prick off the Figure 6, and ſeek how often 8758 in 40087, which will be four times; put 4 in the Quotient, and multiply, ſaying, 4 times 6 (the Figure laſt pricked off) is 24, for which I carry 2, and ſay 4 times 8 is 32, and 2 I carry is 34; ſet down 4, and carry 3; multiply the reſt of the Figures, and ſubtract as before, and ſo proceed after the ſame manner, until all the Figures of the Diviſor be pricked off, to the laſt Figure. See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this third Example, the Unit's Place of the Diviſor, falls under 1, the firſt Place of Decimals; therefore the firſt Figure of the Quotient is in the firſt Place of Decimals; ſo the Quotient will be all Decimals. Then, becauſe the Quotient Figures, and the Figures of the Diviſor will be of an equal Number, daſh off the 43 in the Diviſor, and the 7 in the Dividend, as uſeleſs, and divide as before.

217·35|43

```
217·35|43)25·136|7(.11564
 . ..      21735
           -----
            3401
            2174
            ----
            1227
            1087
            ----
             140
             130
             ---
              10
               8
              --
               2
```

Altho' I have hitherto given Directions for proportioning the Divisor and Dividend, so as to bring into the Quotient what Number of Decimals you please, yet there is no absolute Necessity for it; but you may carry on your Division to what Degree you please, before you begin to prick off the Figures of the Divisor, in order to contract the Work, as in the following Examples, where it is not required to prick off any determinate Number of Decimals, but it may be done according to Discretion.

```
2.756756)7414.76717(2689.67118
          5513512
          -------
          19012551
          16540536
          --------
          24720157
          22054048
          --------
           2666109
           2481080
           -------
            185029
            165405
            ------
             19624
             19297
             -----
               327
               276
               ---
                51
                28
                --
                23
                22
                --
                 1
```

12.34254

```
12.34254)514.75498(41.705757
    ...     4937016
            -------
            2105338
            1234254
            -------
             871084
             863978
             ------
               7106
               6171
               ----
                935
                864
                ---
                 71
                 62
                 --
                  9
                  8
                  -
                  1
```

CHAP. VII.

Extraction of the SQUARE ROOT.

If a Square Number be given;

TO find the Root thereof, that is, to find out ſuch a Number, as being multiplied into itſelf, the Product ſhall be equal to the Number given; ſuch Operation is called, *The Extraction of the Square Root*; which to do, obſerve the following Directions.

1*ſt*,

1st, You must point your given Number; that is, make a Point over the Unit's Place, another upon the Hundred's, and so upon every second Figure throughout.

2dly, Then seek the greatest square Number in the first Period towards the Left hand, placing the square Number under that Point, and the Root thereof in the Quotient, and subtract the said square Number from the first Point, and to the Remainder bring down the next Point, and call that the Resolvend.

3dly, Then double the Quotient, and place it, for a Divisor, on the Left-hand of the Resolvend; and seek how often the Divisor is contained in the Resolvend (reserving always the Unit's Place), and put the Answer in the Quotient, and also on the Right-hand Side of the Divisor; then multiply by the Figure last put in the Quotient, and subtract the Product from the Resolvend (as in common Division), and bring down the next Point to the Remainder (if there be any more), and proceed as before.

A Table *of* Squares *and* Cubes, *and their* Roots.

Root	1	2	3	4	5	6	7	8	9
Square	1	4	9	16	25	36	49	64	81
Cube	1	8	27	64	125	216	343	512	729

Example 1. Let 4489 be a Number given, and let the square Root thereof be required.

4489(67
36

127)889 Resolvend.
.889 Product.

...

First,

First, Point the given Number, as before directed; then, by the little Table aforegoing, seek the greatest square Number in 44 (the first Point to the Left-hand), which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the Right-hand, so it makes 889 for a Resolvend; then double the Quotient 6, and it makes 12; which place on the Left-hand for a Divisor, and seek how often 12 in 88 (reserving the Unit's Place), the Answer is 7 times; which put in the Quotient, and also on the Right-hand Side of the Divisor, and multiply 127 by 7, as in common Division, and the Product is 889, which subtracted from the Resolvend, there remains nothing; so is your Work finished; and the square Root of 4489 is 67; which Root if you multiply by itself, that is 67 by 67, the Product will be 4489, equal to the given square Number, and prove the Work to be right.

Example 2. Let 106929 be a Number given, and let the square Root thereof be required.

```
     .  .  .
     106929(327
      9
     -------
  62)169 Resolvend.
     124 Product.
     -------
 647)4529 Resolvend.
     4529 Product.
     -------
     ....
```

First, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thousands; then seek what is the greatest square Number in 10 (the first Point), which by the little

Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then ſubtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Reſolvend; then double the Quotient 3, and it makes 6, which place on the Left-hand of the Reſolvend for a Diviſor, and ſeek how often 6 in 16; the Anſwer is twice; put 2 in the Quotient, and alſo on the Right-hand of the Diviſor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which ſubtract from the Reſolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Reſolvend. Then double the Quotient 32, and it makes 64, which place on the Left Side of the Reſolvend for the Diviſor, and ſeek how often 64 in 452, which you'll find 7 times; put 7 in the Quotient, and alſo on the Right-hand of the Diviſor, making it 647, which multiplied by the 7 in the Quotient, it makes 4529, which ſubtracted from the Reſolvend, there remains nothing. So 327 is the Square Root of the given Number.

Example 3. Let 2268741 be a ſquare Number given, the Root whereof is required.

```
          . . . .
          2268741(1506.23
          1
          -------
        25)126
           125
          -------
      3006)18741
           18036
          -------
     30122)70500
           60244
          -------
    301243)1025600
           903729
          -------
Remains    121871
```

Having

Having pointed the given Number as before directed, seek what is the greatest square Number in the first Point 2, which is 1; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; subtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and set it on the Right-hand, making it 126; double the 1 in the Quotient; which makes 2; set 2 on the Left-Hand for a Divisor, and ask how often 2 in 12, which will be 5 times; put 5 in the Quotient, and also on the Right-hand of the Divisor, making it 25; multiply (as in common Division) 25 by 5, and subtract the Product, 125, from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Resolvend; and double the 15 in the Quotient, it makes 30 for a new Divisor. Then seek how often 30 in 18, which you can't have; so that you must put 0 in the Quotient, and also on the Right-hand of the Divisor, and bring down the next Point, and it makes 18741 for another new Resolvend. Then seek how often 300 in 1874, which will be six times; put 6 in the Quotient, and also on the Right-hand of the Divisor; multiply and subtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a time, to the Remainders, and so prosecute the Work to what Number of Decimal Parts you please; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Divisor: Then seek how often 3012 in 7050 (rejecting the Unit's Place), which will be twice; put 2 in the Quotient, and also on the Right-hand of the Divisor, and multiply and subtract as before, and the Remainder will be 10256; to which annex two Cyphers, and proceed as before, and you will get a 3 in the Quotient next. So the square Root of the given Number is 1506.23, which being squared, or multiplied by itself, and the last Remainder added, will make the given Number as follow:

```
                 1506.23
                 1506.23
                ---------
                  451869
                 301246
                903738
               7531150
              150623
              ------------
              2268728.8129
The Remainder add   12.1871
              ------------
        Proof 2268741.0000
```

Some more Examples for Practice.

```
               . . . .
*Example* 1.   7596796(2756.228  Root.
               4
               ---
           47)359
              329
              ---
          545)3067
               2725
               ----
         5506)34296
               33036
               -----
        55122)126000
                110244
                ------
       551242)1575600
                1102484
                -------
      5512448)47311600
                44099584
                --------
                 3212016
```

Example

```
Example 2.   751417.5745(866.84  Root.
             64
             ----
       166)1114
            996
          -----
      1726)11817
           10356
           ------
     17328)146157
            138624
            ------
      173364)753345
             693456
             ------
              59889
```

If the given Number be a mixed Number, *viz.* consisting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4, 6, 8, &c. that so there may a Point fall upon the Unit's Place of the whole Numbers, as in this last Example, and in that following.

Example 3. Let 656714.37512 be given, to find the square Root.

```
           . .  . .  . .  .
          656714.375120(810.379 Root.
          64
          ——
     161)167
         161
         ———
   16203)61437
          48609
          —————
  162067)1282851
         1134469
         ———————
 1620749)14838220
         14586741
         ————————
Remains    251479
```

In this Example there are five Places of Decimals; therefore put a Cypher to it, to make them even, that so there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a Decimal Fraction, the Work differs nothing from the Examples aforegoing, only you must be mindful to point your given Number right; for (as was before directed) the Number of Places must always be made even, and then begin to point at the Right-hand, as in whole Numbers.

If it be a Vulgar Fraction, it must be reduced to a Decimal, by the first Rule of the second Chapter.

I shall give an Example or two in each Case, and so conclude this Chapter.

Let

Let .125 be a Decimal Fraction given, whose square Root is required; and let it be required to have four Places of Decimals in the Root.

```
   . . . .
 .12500000(.3535
   9
  ----
 65)350
    325
  ----
703)2500
    2109
   -----
7065)39100
     35325
    ------
      3775
```

In this Example there must be five Cyphers annexed, because two Places in the Square make but one in the Root.

Let the square Root of .00715 be required.

```
   . . .
 .007150(.084
  64
  -----
164)750
    656
    ---
     94
```

In this a Cypher is added to make the Places even.

Let $\frac{7}{8}$ be a Vulgar Fraction given, whose square Root is required.

	• • • •
8)7000	(87500000(.9354
64	81
60	183)650
56	549
40	1865)10100
40	9325
..	18704)77500
	74816
	2684

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875; to which annex Cyphers, and extract the square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{3}{960}$ be a Vulgar Fraction, whose square Root is required.

	• • • •
96|0)3.000000	(.00312500(.0559 Root.
288	25
120	105)625
96	525
240	1109)10000
192	9981
480	19
480	
...	

In

In extracting the Root of this, becauſe the firſt Point conſiſts of Cyphers, there muſt be a Cypher put firſt in the Quotient.

To prove this Rule, ſquare the Root, and to the Product add the Remainder, as was before directed. To ſquare a Number, is to multiply it by itſelf; and to cube it, is to multiply the Square of the Number by the Number itſelf.

CHAP. VIII.

Extraction of the CUBE ROOT.

TO extract the Cube Root, is nothing elſe but to find ſuch a Number, as being firſt multiplied into itſelf, and then into that Product, produceth the given Number; which to perform, obſerve the following Directions.

1*ſt*, You muſt point your given Number, beginning with the Unit's Place, and make a Point, or Dot, over every third Figure towards the Left-hand.

2*dly*, Seek the greateſt Cube Number in the firſt Point, towards the Left-hand, putting the Root thereof in the Quotient, and the ſaid Cube Number under the firſt Point, and ſubtract it therefrom, and to the Remainder bring down the next Point, and call that the Reſolvend.

3*dly*, Triple the Quotient, and place it under the Reſolvend; the Unit's Place of this under the Ten's Place of the Reſolvend; and call this the Triple Quotient.

4*thly*, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of

this under the Ten's Place of the triple Quotient, and call this the Triple Square.

5*thly*, Add theſe two together, in the ſame Order as they ſtand, and the Sum ſhall be the Diviſor.

6*thly*, Seek how often the Diviſor is contained in the Reſolvend, rejecting the Unit's Place of the Reſolvend (as in the Square Root), and put the Anſwer in the Quotient.

7*thly*, Cube the Figure laſt put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Reſolvend.

8*thly*, Multiply the Square of the Figure laſt put in the Quotient, into the triple Quotient, and place the Product under the laſt, one Place more to the Left-hand.

9*thly*, Multiply the triple Square by the Figure laſt put in the Quotient, and place it under the laſt, one Place more to the Left-hand.

10*thly*, Add the three laſt Numbers together, in the ſame Order as they ſtand, and call that the Subtrahend.

Laſtly, Subtract the Subtrahend from the Reſolvend, and if there be another Point, bring it down in the Remainder, and call that a new Reſolvend, and proceed in all Reſpects as before.

Example

Example 1. Let 314432 be a Cubic Number, whose Root is required.

```
 .  .
314432(68 Root.
216
-----------
 98432 Resolvend.
 ---------
    18 Triple Quotient of 6.
   108 Triple Square of the Quotient 6.
  ----
  1098 Divisor.
  ----
    512 Cube of 8, the last Figure of the Root.
   1152 The Square of 8, by the triple Quotient.
   864  The triple Square of the Quotient 6 by 8.
  ------
  98432 The Subtrahend.
  ------
  .....
```

After you have pointed the given Number, seek what is the greatest Cube Number in 314, the first Point, which, by the former little Table, (*Page* 34), you will find to be 216, which is the nearest that is less than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; so will it make 98432 for the Resolvend. Then triple the Quotient 6, it makes 18, which write down the Unit's Place, 8, under 3, the Ten's Place of the Resolvend. Then square the Quotient 6, and triple the Square, and it makes 108, which write under the triple Quotient, one Place toward the Left-hand; then add those two Numbers together, and they make 1098 for the Divisor. Then seek how often the Divisor is contained in the Resolvend, (rejecting the Unit's Place thereof), that is, how often 1098 in 9843, which is 8 times;

8 times; put 8 in the Quotient, and the Cube thereof below the Divisor, the Unit's Place under the Unit's Place of the Resolvend. Then square the 8 last put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient, 18; the Product is 1152; set this under the Cube of 8, the Units of this under the Tens of that. Then multiply the triple Square of the Quotient by 8, the Figure last put in the Quotient, the Product is 864; set this down under the last Product, a Place more to the Left-hand. Then draw a Line under those three, and add them together, and the Sum is 98432, which is called the Subtrahend; which being subtracted from the Resolvend, the Remainder is nothing; which shews the Number to be a true cubic Number, whose Root is 68; that is, if 68 be cubed, it will make 314432.

For if 68 be multiplied by 68, the Product will be 4624; and this Product, multiplied again by 68, the last Product is 314432, which shews the Work to be right.

	68
	68
	544
The Work	408
	4624
	68
	36992
	27744
The Proof	314432

Example 2. Let the Cube Root of 5735339 be required.

After you have pointed the given Number, seek what is the greatest Cube Number in 5, the first Point, which (by the little Table, *Page* 34) you will find to be 1; which place under 5, and 1, the Root thereof, in

in the Quotient; and ſubtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Reſolvend. Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which ſet one under another, in their Order, and added, makes 33 for the Diviſor. Seek how often the Diviſor in the Reſolvend, and proceed as in the laſt Example.

5735339)179	Root.
1	
4735	
3	The Triple of the Quotient 1, the firſt Figure.
3	The triple Square of the Quotient 1.
33	The Diviſor.
343	The Cube of 7, the ſecond Figure of the Root.
147	The Square of 7, multipl. in the triple Quot. 3.
21	The triple Square of the Quot. multiplied by 7.
3913	The Subtrahend.
822339	The new Reſolvend.
51	The Triple of the Quot. 17, the two firſt Fig.
867	The triple Square of the Quotient 17.
8721	Diviſor.
729	The Cube of 9, the laſt Figure of the Root.
4131	The Squ. of 9, multipl. by the triple Quot. 51.
7803	The triple Square of the Quotient 867 by 9.
822339	The Subtrahend.

In

In this Example, 33, the first Divisor, seems to be contained more than seven times in 473, the Resolvend, after the Unit's Place has been rejected; but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Resolvend.

Some more Examples for Practice.

32461759(319 Root.
27

5461 Resolvend.

9 The Triple of 3.
27 The triple Square of 3.

279 The Divisor.

1 The Cube of 1, the second Figure.
9 The triple Quotient, by the Square of 1.
27 The triple Square, multiplied by 1, the 2d Fig.

2791 The Subtrahend.

2670759 A new Resolvend.

93 The Triple of 31.
2883 The triple Square of 31.

28923 The Divisor.

729 The Cube of 9, the last Figure.
7533 The Square of 9, by 93 the triple Quotient.
25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

....

864

84604519(439 Root.
64

20604 Resolvend.

12 Triple of 4.
48 Triple Square of 4.

492 Divisor.

27 Cube of 3.
108 Square of 3, by the triple Quotient.
144 Triple Square by 3.

15507 Subtrahend.

5097519 Resolvend.

129 Triple of 43.
5547 Triple Square of 43.

55599 Divisor.

729 Cube of 9.
10449 Square of 9 by 129.
49923 Triple Square by 9.

5097519 Subtrahend.

....

259697989(638
216

43697	Resolvend.
18	Triple of 6.
108	Triple Square of 6.
1098	Divisor.
27	Cube of 3, the second Figure.
162	Square of 3 by 18.
324	Triple Square, 108, by 3.
34047	Subtrahend.
9650989	Resolvend.
189	Triple of 63.
11907	Triple Square of 63.
119259	Divisor.
512	Cube of 8.
12096	Square of 8 by 189.
95256	Triple Square, 11907, by 8.
9647072	Subtrahend.
3917	Remainder.

25917056)295·9	
8	
17917	Resolvend.
6	Triple of 2.
12	Triple Square of 2.
126	Divisor.
729	Cube of 9, the 2d Figure.
486	Square of 9 by 6.
108	Triple Square by 9.
16389	Subtrahend.
1528056	Resolvend.
87	Triple of 29.
2523	Triple Square of 29.
25317	Divisor.
125	Cube of 5, the 3d Figure.
2175	Square of 5 by 87.
12615	Triple Square by 5.
1283375	Subtrahend.
244681000	Resolvend.
885	Triple of 295.
261075	Triple Square of 295.
2611635	Divisor.
729	Cube of 9, the last Figure.
71685	Square of 9 by 885.
2349675	Triple Square by 9.
235685079	Subtrahend.
8995921	Remainder.

In this Example I annex 3 Cyphers to the Remainder, which makes the 3d Resolvend; by which means I bring one Place of Decimals. And so you may proceed to more decimal Places at Pleasure, by annexing three Cyphers to the next Remainder, and carrying on the Work as before.

. . . .	
22069810125(2805	
8	
14069	Refolvend.
6	Triple of 2.
12	Triple Square by 2.
126	Divifor.
512	Cube of 8.
384	Square of 8 by 6.
96	Triple Square by 8.
13952	Subtrahend.
117810125	New Refolvend.
84	Triple of 28.
2352	Triple Square of 28.
23604	Divifor.
840	Triple of 280.
235200	Triple Square of 280.
2352840	New Divifor.
125	Cube of 5.
21000	Square of 5 by 840.
1176000	Triple Square by 5.
117810125	Subtrahend.
. . . .	

In this Example 13952, being fubtracted from the Refolvend 14069, the Remainder is 117; to which bring down 810, the 3d Point, and it makes 117810, for a new Refolvend; and the next Divifor is 23604, which you cannot have in the faid Refolvend (the Unit's Place being rejected); fo you muft put 0 in the Quotient, and feek a new Divifor (after you have brought down your laft Point to the Refolvend); which new Divifor is 2352840; and you will find it to be contained 5 times. So proceed to finifh the reft of the Work.

93759·575070(45·42	
64	
29759	Resolvend.
12	Triple of 4, the first Figure.
48	Triple Square of 4.
492	Divisor.
125	Cube of 5, the 2d Figure.
300	Square of 5 by 12, the triple Quotient.
240	Triple Square by 5.
27125	Subtrahend.
2634575	Resolvend.
135	Triple of 45.
6075	Triple Square of 45.
60885	Divisor.
64	Cube of 4.
2160	Square of 4 by 135.
24300	Triple Square by 4.
2451664	Subtrahend.
182911070	Resolvend.
1362	Triple of 45·4.
618348	Triple Square of 45·4.
6184842	Divisor.
8	Cube of 2.
5448	Square of 2 by 1362.
1236696	Triple Square by 2.
123724088	Subtrahend.
59186982	Remainder.

In extracting the Cube Root of a mixed Number, always obſerve to make the Decimal Part conſiſt of either three, ſix, nine, &c. Places; that is, always to conſiſt of even Points, as in the laſt Example, where the Decimal Places were five; to which I annexed a Cypher to make up ſix, and ſo I proceed to point it; and by that means I have a Point fall upon the Unit's Place of whole Numbers, which you muſt always obſerve.

To extract the Cube Root of a Fraction.

This is done in the ſame manner as in whole Numbers, if the foregoing Directions are obſerved, for the true pointing the Number; for, as was before directed, the Decimal muſt always conſiſt of three, ſix, nine, &c. Places; and if it be not ſo, it muſt be made ſo, by annexing Cyphers, as is ſaid above.

If the Cube Root of a vulgar Fraction be required, you muſt firſt reduce it to a Decimal, and then extract the Root.

Examples of each follow.

Example

Example 1. Let the Cube Root of .401719179 be required.

.401719179(.737	Root.
343	
58719	Resolvend.
21	Triple of 7.
147	Triple Square of 7.
1491	Divisor.
27	Cube of 3.
189	Square of 3 by 21.
441	Triple Square by 3.
46017	Subtrahend.
12702179	Resolvend.
219	Triple of 73.
15987	Triple Square of 73.
160089	Divisor.
343	Cube of 7.
10731	Square of 7 by 219.
111909	Triple Square by 7.
11298553	Subtrahend.
1403626	Remainder.

Example

Example 2. Let the Cube Root of .0001416 be required.

```
 .000141600(.052 Root.
  125
  ------
   16600 Resolvend.
   -----
      15  Triple of 5.
     75   Triple Square of 5.
    ----
    765   Divisor.
    ----
       8  Cube of 2.
      60  Square of 2 by 15.
    150   Triple Square by 2.
   ------
   15608  Subtrahend.
   ------
     992  Remainder.
```

Example 3. Let $\frac{5}{276}$ be a Vulgar Fraction, whose Cube Root is required.

By the first Rule of Chapter II. reduce the Vulgar Fraction to a Decimal.

```
276)5.000000000(.018115942
      ........
      --------
      2240
        320
         440
        1640
         2600
          1160
            560
            ---
              8
```

.01811

```
 .  .  .
.018115942(.262 Root.
  8
 -----
 10115 Resolvend.
 -----
    6  Triple of 2.
   12  Triple Square of 2.
  ----
  126  Divisor.
  -----
   216 Cube of 6.
  216  Square of 6 by the Triple of 2.
  72   Triple Square by 6.
  -----
  9576 Subtrahend.
  -----
   539942 Resolvend.
   ------
       78 Triple of 26.
     2028 Triple Square of 26.
    ------
    20358 Divisor.
    ------
        8 Cube of 2.
      312 Square of 2 by 78.
    4056  Triple Square 2028 by 2.
   ------
   408728 Subtrahend.
   ------
   131214 Remainder.
```

You may prove the Truth of the Work, by cubing the Root found, as was ſhewed in the firſt Example; and if any thing remains, add it to the ſaid Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will

I will ſhew the Proof of the fifth Example (*Page* 50), the given Number being 259697989, whoſe Root is 638, it being a ſurd Number, there remains 3917.

	638
	638
	5104
	1914
	3828
The Square	407044
	638
	3256352
	1221132
	2442264
The Cube	259694072
Remainder add	3917
Proof equal to the given Numb.	259697989

CHAP. IX.

Multiplication of Feet, Inches, and Parts.

IN the multiplying of Feet, Inches, *&c.* I ſhall endeavour to lay down ſuch eaſy and familiar Rules, as may eaſily be underſtood by the meaneſt Capacity.

Example.

Example 1. Let 7 Feet 9 Inches be multiplied by 3 Feet 6 Inches.

F.	I.	
7	9	
3	6	
23	3	*Pts.*
3	10	6
27	1	6

Firſt, Multiply 9 Inches by 3, ſaying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; ſet down 3 under Inches, and carry 2 to the Feet, ſaying, 3 times 7 is 21, and 2 that I carry make 23; ſet down 23 under the Feet.

Then begin with 6 Inches, ſaying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; ſet down 6 Parts, and carry 4, ſaying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which ſet down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiplied by 9 Feet 8 Inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

Firſt, Multiply by 9 Feet, ſaying, 9 times 7 is 63, which is 5 Feet 3 Inches; ſet down 3, and carry 5, ſaying, 9 times 5 is 45, and 5 I carry is 50; ſet down 0, and carry 5, ſaying, 9 times 7 is 63, and 5

is 68; ſet down 68, and proceed to multiply by 8 Inches, ſaying, 8 times 7 is 56; the Twelves in 56 are four times, and 8 remains; ſet 8 a Place to the Right-hand, and carry 4: Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 remains; ſet down 50 Feet 4 Inches, and add all up together, and you will find the Product 730 Feet 7 Inches 8 Parts.

I will repeat the laſt Example again, and ſhew another Way to work it, which, I think, is better, and more expeditious, when there are more Figures than one in the Feet; thus,

F.	I.	
75	7	
9	8	
680	3	
25	2	4
25	2	4
730	7	8

Multiply by 9 Feet, firſt, as above directed; then, inſtead of multiplying by 8 Inches, let the Inches be parted into ſuch aliquot or even Parts of a Foot, as you find to be contained in that Figure; if you take ſuch Parts of the Multiplicand, and add them to the former Product, the ſum will give the Anſwer: Thus, 8 Inches may be parted into four, and 4, becauſe 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and ſet it down twice and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the ſame as before; thus, ſay how often 3 in 7, which is twice; ſet down 2; then, becauſe twice 3 is 6, ſay, 6 out of 7, and there remains 1, for which you muſt add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times; ſet down 5; and, becauſe 3 times 5 is 15, there is 0 remains. Then go to the

7

7 Inches, ſaying, the Threes in 7 are twice; ſet down 2 in the Inches; and becauſe twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet 2 Inches 4 Parts; which ſet down again, and add all together, the Sum is 730 Feet 7 Inches 8 Parts; the ſame as before.

Example 3. Let 97 Feet 8 Inches be multiplied by 8 Feet 9 Inches.

F.	I.
97	8
8	9
781	4
48	10
24	5
854	7

Begin, firſt, to multiply by 8 Feet, ſaying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; ſet down 4 Inches, and carry 5, ſaying, 8 times 7 is 56, and 5 I carry is 61; ſet down 1, and carry 6, ſaying, 8 times 9 is 72, and 6 I carry is 78, which ſet down: Then, inſtead of multiplying by 9 Inches, take the aliquot Parts of 12 which 9 makes, which is 6 and 3; 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet 8 Inches, which is 48 Feet 10 Inches; and becauſe 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches. See the Work, as above.

Example 4. Let 75 Feet 9 Inches be multiplied by 17 Feet 7 Inches.

F.	I.	
75	9	
17	7	
525		
75		
25	3	P.
18	11	3
8	6	
4	3	
1331	11	3

In this Example, becauſe there are more than 12 Feet in the Multiplier, therefore I firſt multiply the 75 by 17 Feet; then, becauſe the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet 11 Inches 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches (not meddling with the 7 Inches, becauſe that was multiplied into the 9 before); then add all theſe together, and the Sum is 1331 Feet 11 Inches 3 Parts.

Example 5. Let 87 Feet 5 Inches be multiplied by 35 Feet 8 Inches.

F.	I.	P.
87	5	
35	8	
435		
261		
29	1	8
29	1	8
11	8	0
2	11	0
3117	10	4

Work here as in the laſt Example. After you have multiplied the Feet, then take the aliquot Parts of 8 Inches, which are two Thirds; therefore take the third Part of 87 Feet 5 Inches, and ſet it down twice. Thus the third Part of 87 Feet 5 Inches is 29 Feet 1 Inch 8 Parts; ſet this down twice; then the aliquot Parts of 5 Inches are 4 and 1, that is, a third Part and a 12th Part; therefore take a third Part of 35, which is 11 Feet 8 Inches, and a 12th Part of 35 is 2 Feet 11 Inches; ſet all theſe one under another, and add them together, and the Sum is 3117 Feet 10 Inches 4 Parts.

Example 6. Let 259 Feet 2 Inches be multiplied by 48 Feet 11 Inches.

F.	I.	P.
259	2	
48	11	
2072		
1036		
129	7	
86	4	8
21	7	2
8	0	0
12677	6	10

Firſt, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, a half, a third, and a twelfth; therefore take the half of 259 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet 4 Inches 8 Parts, and the twelfth Part of 259 Feet 2 Inches is 21 Feet 7 Inches 2 Parts; or (becauſe 1 is the fourth Part of 4), you may more readily take the fourth Part of 86 Feet 4 Inches 8 Parts, which is alſo 21 Feet 7 Inches 2 Parts; then 2 Feet are the ſixth of 12, take the ſixth of 48 Feet, which will be 8 Feet, which place under the Feet; then add all together, and the Sum is 12677 Feet 6 Inches 10 Parts. See the foregoing Work.

I ſhall ſet down only the working of ſome few Examples in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, *&c.*

	F.	I.	P.
	179	3	
	38	10	
	1432		
	537		
	89	7	6
	59	9	0
	3	6	0
Product	6960	10	6

	F.	I.	P.
	246	7	
	46	4	
	1476		
	984		
	82	2	4
	15	4	0
	11	6	0
Product	11425	0	4

F.	I.	P.
246	7	
36	9	
1476		
738		
123	3	6
61	7	9
12	0	0
9	0	0
9061	11	3

	F.	I.	P.
	257	9	
	39	11	
	2313		
	771		
	128	10	6
	85	11	0
	25	5	9
	19	6	0
	9	9	0
Product	10288	6	3

Example

Example 11. Let 7 Feet 5 Inches 9 Parts be multiplied by 3 Feet 5 Inches 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
22	5	3	S.	
3	1	4	9	T.
	1	10	5	3
25	8	6	2	3

In this Example, I firſt begin with 3 Feet, and thereby multiply 7 Feet 5 Inches 9 Parts: Firſt, I ſay, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; ſet down 3 under the Parts, and carry 2, ſaying 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; ſet down 5 Inches, and carry 1, and ſay, 3 times 7 is 21, and 1 I carry is 22; ſet down 22 Feet: Then begin with 5 Inches, ſaying 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; ſet down 9 Seconds a Place towards the Right-hand, and carry 3 Parts, ſaying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; ſet down 4 Parts, and carry 2, ſaying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; ſet down 3 Feet 1 Inch, and begin to multiply by 3 Parts, ſaying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; ſet down 3 Thirds, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; ſet down 5 Seconds, and carry 1, ſaying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts, which ſet down, and add all up, and the Product is 25 Feet 8 Inches 6 Parts 2 Seconds 3 Thirds.

Note, That in multiplying Feet, Inches, and Parts, *&c.* if Feet be multiplied by Feet, the Product is Feet; and Feet multiplied by Inches, the Product is Inches;

and the twelfth Part is Feet; and Parts multiplied by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiplied by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiplied by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds. So that if you begin to multiply Parts by Feet in the first Row, and Parts by Inches in the second Row, and Parts by Parts in the third Row, the first Figures in every Row will stand a Place more towards the Right-hand, as you may see in the last Example.

Example 12. Let 37 Feet 7 Inches 5 Parts be multiplied by 4 Feet 8 Inches 6 Parts.

F.	I.	P.		
37	7	5		
4	8	6		
150	5	8	S.	
12	6	5	8	
12	6	5	8	T.
1	6	9	8	6
177	1	5	0	6

First, I multiply by 4 Feet, saying, 4 times 5 is 20, which is 1 Inch 8 Parts; set down 8, and carry 1, saying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet 5 Inches; set down 5 Inches, and carry 2, saying, 4 times 7 is 28, and 2 I carry is 30; set down 0, and carry 3, and say, 4 times 3 is 12, and 3 is 15; set down 15. Then I begin with 8 Inches; but, because the Feet in the Multiplicand are more than 12, it will be the best Way to work for the aliquot Parts of 8; so here I work for 4 Inches, and set that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet 7 Inches 5 Parts, which is twelve Feet six Inches five Parts eight Seconds; set this down twice: Then begin with 6 Parts; but, instead of multiplying, take half 37 Feet 7 Inches 5 Parts

5 Parts (becauſe 6 is half 12), and ſet it a Place more to the Right-hand: Thus, the half of 37 Feet is 18, which I muſt count 18 Inches, becauſe the Multiplier is 6 Parts; ſo the half of 37 Feet 7 Inches 5 Parts, is one Foot ſix Inches nine Parts eight Seconds ſix Thirds; which ſet down, and add all up together, and the Sum is 177 Feet 1 Inch 5 Parts 0 Seconds 6 Thirds.

Example 13. Let 311 Feet 4 Inches 7 Parts be multiplied by 36 Feet 7 Inches 5 Parts.

F.	I.	P.	S.	T.
311	4	7		
36	7	5		
1866				
933				
103	9	6	4	
77	10	1	9	
8	7	9	6	4
2	1	11	4	7
12	0	0	0	0
1	0	0	0	0
	9	0	0	0
11402	2	4	11	11.

In this Example, becauſe the Feet both in the Multiplier and Multiplicand are compound Numbers, I firſt multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; ſo take the third Part of 311 Feet 4 Inches 7 Parts, which is 103 Feet 9 Inches 6 Parts 4 Seconds, and the fourth Part is 77 Feet 10 Inches 1 Part 9 Seconds; ſet theſe down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part; ſo the third Part of 311 Feet 4 Inches 7 Parts is 103 Feet 9 Inches 6 Parts 4 Seconds; but, becauſe the Multiplier is Parts, it muſt be ſet a Place

to

to the Right-hand, that is, the 103 must be Inches, which is 8 Feet 7 Inches; therefore I set down 8 Feet 7 Inches 9 Parts 6 Seconds 4 Thirds. Then, because 1 Inch is a fourth Part of 4 Inches, therefore I take a fourth Part of 8 Feet 7 Inches 9 Parts 6 Seconds 4 Thirds, which is 2 Feet 1 Inch 11 Parts 4 Seconds 7 Thirds, which is the same as if I had taken a twelfth Part of 311 Feet 4 Inches 7 Parts. Then for 4 Inches in the Multiplicand, instead of multiplying 36 Feet by it, take a third Part, because 4 Inches is the third Part of 12; so the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a third and a fourth; so the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all these together, and the Sum will be 11402 Feet 2 Inches 4 Parts 11 Seconds 11 Thirds.

Example 14. Let 8 Feet 4 Inches 3 Parts 5 Seconds 6 Thirds, be multiplied by 3 Feet 3 Inches 7 Parts 8 Seconds 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		4	10	6	0	2	6		
			5	6	10	3	8	0	
				1	4	8	6	11	0
Product	27	7	3	5	1	8	8	11	0

In this last Example there is no Difficulty, if you do but observe the former Directions, and set every Row a Place more to the Right-hand.

I ſhall only ſet down the working of ſome few Examples more, and ſo conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S.	
80	4	9	9 T.	
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.	
42	7	8	
7	3	6	
298	5	8	
10	7	11	S.
1	9	3	10
310	10	10	10

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S.	
62	3	10	6 T:	
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	3	6	0	0
1809	1	1	9	6

F.	I.	P.		
259	10	8		
18	5	4		
2072				
259			S.	
86	7	6	8	
21	7	10	8 T.	
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

F.	I.	P.		
267	7	10		
25	9	7		
1335				
534				
133	9	11	S.	
66	10	11	6	
11	1	9	11	T.
1	10	3	7	10
12	6	0	0	0
2	1	0	0	0
1	0	6	0	0
	8	4	0	0
6905	0	5	0	10

F.	I.	P.		
317	9	7		
37	5	9		
2219				
951			S.	
105	11	2	4	
26	5	9	7	T.
13	2	10	9	6
6	7	5	4	9
18	6	0	0	0
9	3	0	0	0
1	6	6	0	0
	3	1	0	0
11910	9	11	1	3

THE

THE
Complete Meaſurer, &c.

PART II.

CHAP. I.

Menſuration of SUPERFICIES.

Superficial Figures are all ſuch as have only Length and Breadth, not having any commenſurable Thickneſs.

§ I. *Of a* SQUARE.

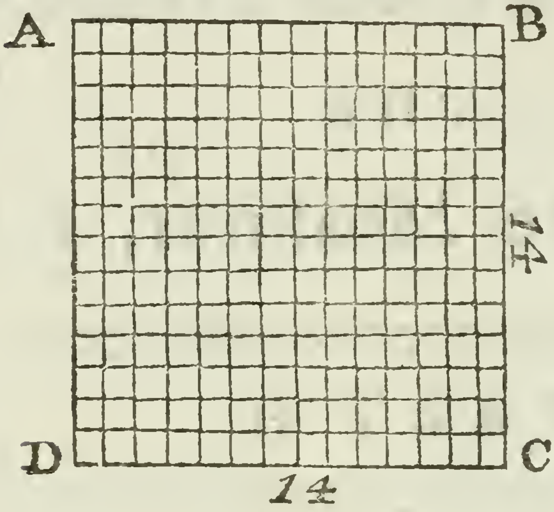
A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or ſquare) Angles. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Side into itſelf, and the Product is the Content.

Let

Let A B C D be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Measure; multiply 14 by itself, and the Product is 196, which is the superficial Content.



$$\begin{array}{r} 14 \\ 14 \\ \hline 56 \\ 14 \\ \hline 196 \end{array}$$ Product.

By Scale and Compasses.

Extend the Compasses from 1, in the Line of Numbers, to 14; the same Extent will reach from the same Point, turned forward to 196.

Demonstration. Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, crossing each other within the Square; so shall the whole great Square be divided into 196 little Squares, as you may see in the Figure above, equal to the Number of Square Feet, Yards, Poles, or other Measures, by which the Side was measured.

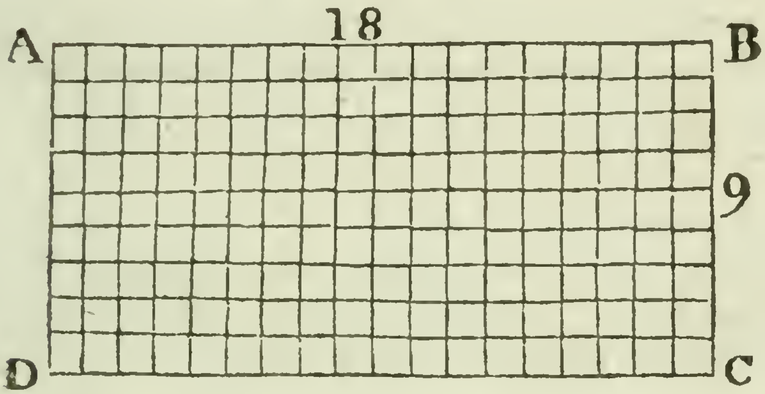
§ II.

§ II. *Of a* PARALLELOGRAM, *or Long Square.*

A Parallelogram is a Figure having four Sides, and as many Right Angles, the oppoſite Sides of it being equal and parallel. To find the ſuperficial Content, this is

The RULE.

Multiply the Length by the Breadth, and the Product is the ſuperficial Content.



Length — 18
Breadth — 9

Product — 162

Let A B C D be a long Square, the Length of it 18 Feet, and the Breadth 9 Feet; which multiplied together, the Product is 162, the ſuperficial Content.

By Scale and Compaſſes.

Extend the Compaſſes in the Line of Numbers from 1 to 9, the ſame Extent will reach from 18 to 162, the ſquare Feet.

Demonſtration. If the Sides AB and CD be each divided into 18 equal Parts, repreſenting 18 Feet; and the Lines AD and BC each divided into 9 equal Parts, and Lines drawn from Point to Point, croſſing each other within the Figure; thoſe Lines will make thereby ſo many little Squares as there are ſquare Feet, *viz.* 162.

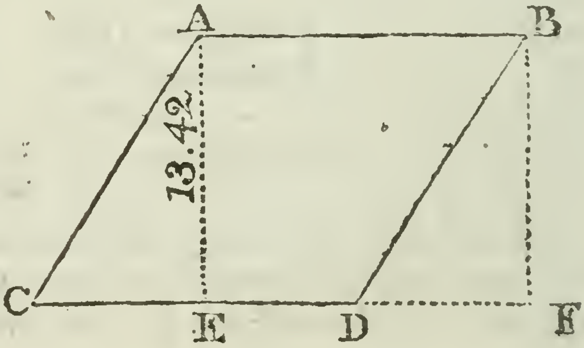
§ III. *Of a* RHOMBUS.

A Rhombus is a Figure repreſenting a Quarry of Glaſs, having four equal Sides, the Oppoſite ones being equal, two Angles being obtuſe, and two acute. To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuſe Angles to the oppoſite Side, and the Product is the Content.

Perpend.	13.42
The Side	15.5
	6710
	6710
	1342
Product	208.010



Let ABCD be a Rhombus given, whoſe Sides are each 15.5 Feet, and the Perpendicular EA is 13.42, which multiplied together, the Product is 208.010; which is the ſuperficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 13.42, that Extent will reach from 15.5, the ſame Way to 208 Feet, the Content.

Demonſtration. Let C D be extended out to F, making DF equal to CE, and draw the Line BF; ſo ſhall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal, and BF is equal to AE, becauſe AB and CF are parallel. Therefore the Parallelogram ABEF is equal to the Rhombus ABCD.

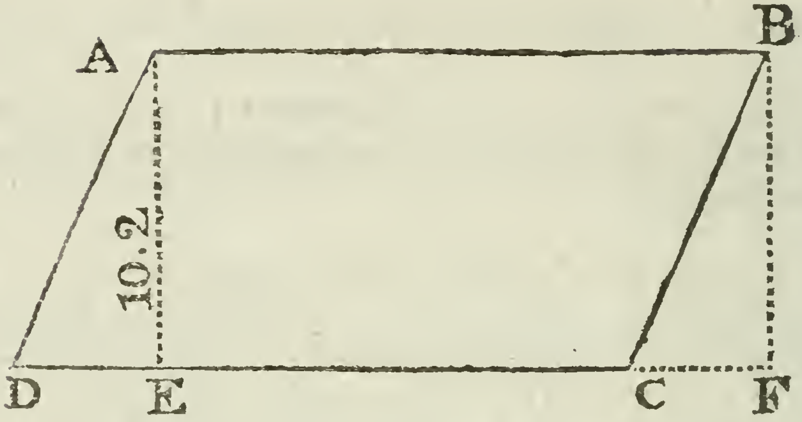
§ IV. *Of a* RHOMBOIDES.

A Rhomboides is a Figure having four Sides, the oppoſite ones being equal and parallel; and alſo four Angles, the oppoſite of which are equal. To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the longeſt Sides by the Perpendicular let fall from one of the obtuſe Angles to one of the longeſt Sides, and the Product is the Content.

$$\begin{array}{r} 19.5 \\ 10.2 \\ \hline 390 \\ 1950 \\ \hline 198.90 \end{array}$$



Let ABCD be a Rhomboides given, whoſe longeſt Sides, AB or CD, is 19.5 Feet, and the Perpendicular AE is 10.2; which multiplied together, the Product is 198.9, that is, 198 ſuperficial Feet and 9 tenth Parts, the Content.

Demonſtration. If DC be extended to F, making CF equal to DE, and a Line drawn from B to F; ſo will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD; which was to be proved.

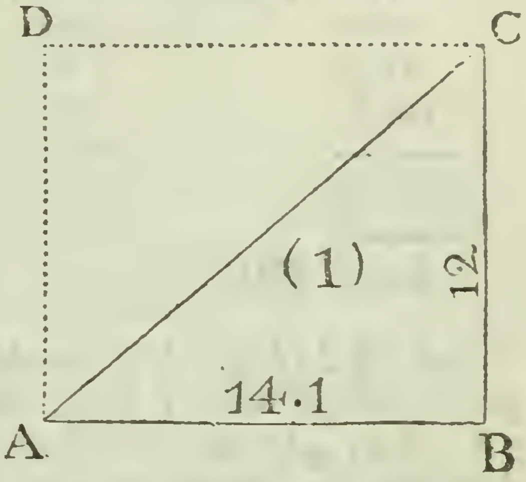
§ V. *Of a* TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are ſuch as have one Right Angle. Oblique-angled Triangles are ſuch as have their Angles either acute or obtuſe. An obtuſe Angle is greater than a Right Angle, that is, it is more than 90 Degrees; and an acute Angle is leſs than a Right Angle. To find the ſuperficial Content thereof, this is

The

The RULE.

Let the Triangle be of what Kind ſoever, multiply the Baſe by half the Perpendicular, or half the Baſe by the whole Perpendicular; or, multiply the whole Baſe by the whole Perpendicular; and take half the Product; any of theſe three Ways will give the Content.

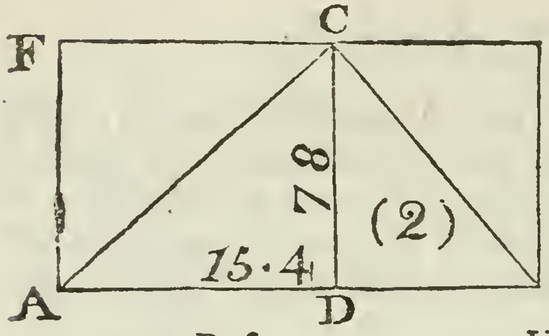


Let A B C be a Right-angled Triangle, whoſe Baſe is 14.1 Feet, and the Perpendicular 12 Feet. Multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or, multiply 14.1 by 12, the Product is 169.2; the half of which is 84.6, the ſame as before.

14.1 Baſe.	14.1 Baſe.
6 Half Perpendicular.	12 Perpendicular.
84.6 Product.	169.2 Product.
	84.6 Half.

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 14.1, that Extent will reach the ſame Way from 12 to 84.6 Feet, the Content.



15·4 Baſe.
3·9 Half Perp.
———
1386
462
———
60.06 Product.

15·4 Baſe.
7.8 Perpend.
———
1232
1078
———
120.12
———
60.06 Half.

7·7 Half Baſe.
7·8 Perpendicular.
———
616
539
———
60.06 Product.

Let ABC (*Fig.* 2.) be an oblique-angled Triangle given, whoſe Baſe is 15·4, and the Perpendicular 7·8; if 15·4 be multiplied by 3·9 (half the Perpendicular), the Product will be 60.06 for the Area, or ſuperficial Content: Or, if the Perpendicular 7·8 be multiplied into half the Baſe 7·7, the Product will be 60.06 as before: Or, if 15·4, the Baſe, be multiplied by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area; the Half thereof is 60,06 Feet, as before. See the Work.

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 15·4, that Extent will reach from 7·8 to 60 Feet, the Content.

Demonſtration. If AD (*Fig.* 1) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC ſhall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram

is

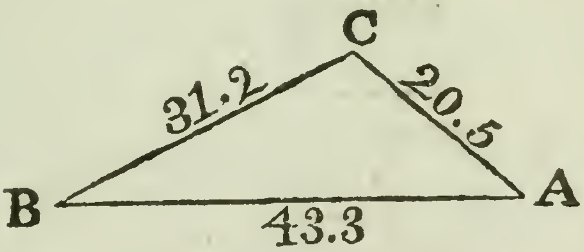
is the Area of the Triangle. In *Fig.* 2, the Parallelogram ABEF is alſo double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle: Which was to be proved.

To find the Area of any plain Triangle by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum: Then ſubtract each Side ſeverally from that half Sum. Which done, multiply that half Sum and the three Differences continually, and out of the laſt Product extract the Square Root; which Square Root ſhall be the Area of the Triangle ſought.

Example. Let ABC be a Triangle, whoſe three Sides are as followeth; *viz.* AB 43.3, AC 20.5, and BC 31.2, the Area is required.



Sides	43.3	4.2	Differences.
	31.2	16.3	
	20.5	27.0	
Sum	95.0		
Half	47.5		

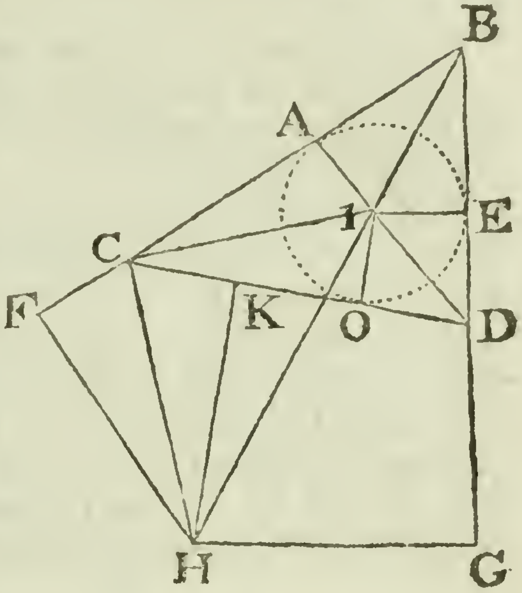
Area

Area 296.31

```
        47.5 The half Sum.
          27 Difference.
        ----
        3325
        950
      ------
      1282.5 Product.
        16.3 Difference.
      ------
       38475
      76950
     12825
    --------
    20904.75 Product.
         4.2 Difference.
    --------
     4180950
    8361900
    --------
    .  .  .  .  .
    87799.9500(296.31
    4
    --
  49)477
     441
    ----
  586)3699
      3516
    ------
 5923)18395
      17769
     ------
59261)62600
      59261
     ------
      3339 Remains.
```

Demon-

Demonstration. In the Triangle BCD, I say, if from the half Sum of the Sides, you subtract each particular Side, and multiply the half Sum and the three Differences together continually, the Square Root of the Product shall be the Area of the Triangle.



First, by the Lines BI, CI, and DI, bisect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is bivided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI, and OI, being all equal to one another, because the Point I is the Center of the inscribed Circle (by *Euclid, Lib.* IV. *Prob.* 4.): Wherefore to the Side BC join CF equal to DE, or DO; so shall BF be equal to half the Sum of the Sides; *viz.* $= \frac{1}{2}$ BC $+ \frac{1}{2}$ BD $+ \frac{1}{2}$ CD.

And BA $=$ BF $-$ CD; for CA $=$ CO and OD $=$ CF; therefore CD $=$ AF; and AC $=$ BF $-$ BD, for BE $=$ BA, and ED $=$ CF; therefore BD $=$ BA $+$ CF, and CF $=$ BF $-$ BC.

Make FH perpendicular to FB, and produce BI to meet it in H. Draw CH, and HK perpendicular to CD. Because the Angle FCK $+$ FHK are equal to two Right Angles (for the Angles F and K are Right Angles) equal also to FCK $+$ ACO (by *Euclid.* I. 13.), and the Angles ACO $+$ AIO are equal to two Right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC are also similar. And the Triangles BAI and BFH are likewise similar.

From

From this Explanation, I fay, the Square of the Area of the given Triangle; that is, BF $q \times$ IA $q =$ BF $\times$ BA $\times$ CA $\times$ CF. In Words:

The Square of BF (the half Sum of the Sides) multiplied into the Square of IA ($=$IE$=$IO) will be equal to the faid half Sum multiplied into all the three Differences.

For IA : BA : : FH : BF; and IA : CF : : AC : FH; becaufe the Triangles are fimilar. By *Euclid, Lib.* VI. *Prop* 4.

Wherefore multiplying the Extremes and Means in both, it will be IA $q \times$ BF $\times$ FH $=$ BA $\times$ CA $\times$ CF $\times$ FH; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be BF $q \times$ IA $q =$ BF $\times$ BA $\times$ CA $\times$ CF. Which was to be demonftrated.

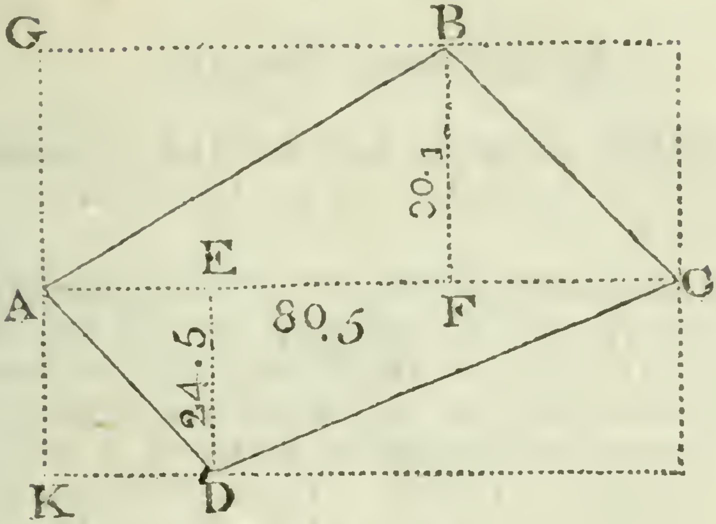
§ VI. *Of a* Trapezium.

A Trapezium is a Figure having 4 unequal Sides, and oblique Angles. To find the Area or fuperficial Content thereof, this is

The RULE.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Areas of the two Triangles, ABC and ACD (by Section V.), and add thofe two Areas together, the Sum fhall be the Area of the Trapezium.

BF$=$30.1



BF = 30.1
DE = 24.5

Sum 54.6

Half 27.3
AC = 80.5

1365
21840

Area 2197.65

Let ABCD be a Trapezium given, the Diagonal of which is 80.5, and the Perpendicular BF 30 1, and the Perpendicular DE 24.5; thefe two added together, the Sum is 54.6, the Half of which is 27.3, and this being multiplied by the Diagonal, 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 40 25, half the Diagonal, be multiplied by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the fame as before.

By

By Scale and Compaffes.

Extend the Compaffes from 2 to 54.6; that Extent will reach from 80.5 to 2197.65, the Area.

Demonftration. This Figure ABCD is compofed of two Triangles, the Triangle ABC is half the Parallelogram AGHC: Alfo the Triangle ACD is equal to half the Parallelogram ACIK, as was proved, Sect. V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area HI = BF + DE; therefore $\frac{1}{2}$ HI × AC (= KI = GH = Area of the Trapezium, which was to be proved.

§ VII. *Of* Irregular Figures.

IRregular Figures are all fuch as have more Sides than four, and the Sides and Angles unequal. All fuch Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of fuch Figures, they muft be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another; and fo find the Areas of the Trapeziums and Triangles feverally, and then add all the Areas together; fo will you have the Area of the whole Figure.

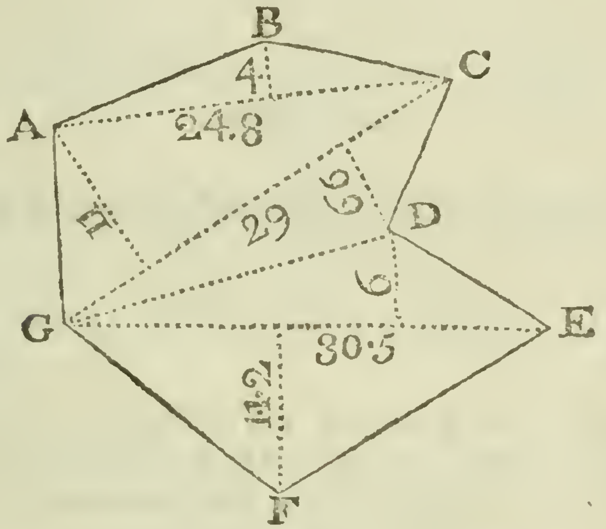
Let ABCDEFG be an irregular Figure given to be meafured; firft, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Area feverally.

Firft, I multiply the Bafe AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then

Then for the Trapezium ACDG, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half of which is 8.8, multiplied by 29, the Diagonal; the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2; the Half is 8.6; which multiplied by 30.5, the Diagonal, the Product is 262.3, the Area. All theſe Areas added together, make 567.1, and ſo much is the Area of the whole irregular Figure. See the Work.



24 8 Baſe AC.
 2 Half Perpendicular.

49.6 Area of ABC.

11 Perpendicular.
6.6

17.6 Sum.

8.8 Half.
29 Diagonal CG.

792
176

255.2 Area of ACGD.

11.2 } Perpendiculars.		30.5	
6 }		8.6	
17.2 Sum.		1830	
		2440	
8.6 Half Sum.			
		262.30	Area of GDEF.
		255.2	Area of ACGD.
		40.6	Area of ABC.
		567.1	Sum of the Areas.

This Figure being compoſed of Triangles and Trapeziums, and thoſe Figures being ſufficiently demonſtrated in the Vth and VIth Sections aforegoing, it will be needleſs to mention any thing of the Demonſtration in this Place.

§ VIII. *Of* REGULAR POLYGONS.

REgular Polygons are all ſuch Figures as have more than four Sides, all the Sides and Angles being equal. Polygons are denominated from the Number of their Sides and Angles.

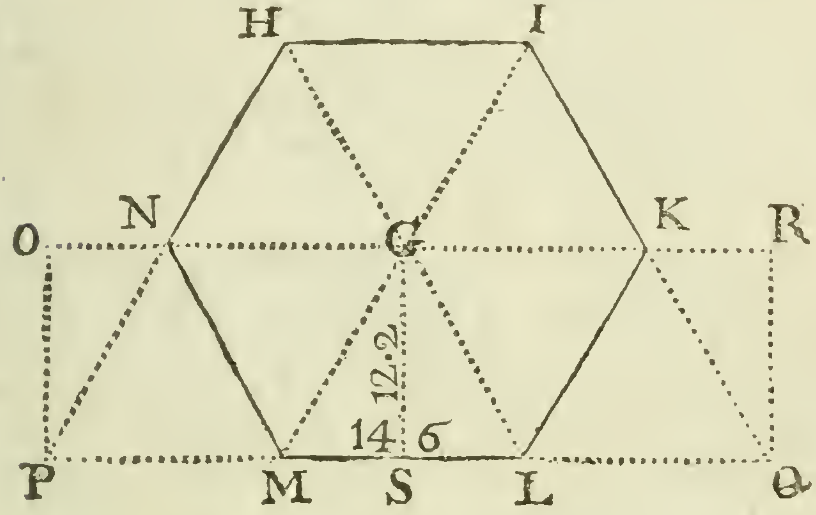
If the Figure conſiſts of	5	Equal Sides and Angles, it is called a Regular	Pentagon.
	6		Hexagon.
	7		Heptagon.
	8		Octagon.
			Enneagon.
	10		Decagon.
	11		Endecagon.
	12		Dodecagon.

To

To find the Area or ſuperficial Content of any regular Polygon, this is

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular let fall from the Center to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



14.6			
3			
43.8	Half Sum of the Sides.		
12.64	The Perpendicular.		
43.8	Half Sum.		
		14.6	
10112		6	
37912			
5056		87.6	Sum of the Sides.
		6.32	Half Perpend.
553.632			
		1752	
		2628	
		5256	
		553.632	Area.

I 2 Let

Let HIKLMN be a regular Hexagon, each Side being 14.6, the Sum of all the Sides is 87.6, the half Sum is 43.8, which multiplied by the Perpendicular GS 12.64, the Product is 553.632: Or if 87.6, the whole Sum of the Sides, be multiplied by half the Perpendicular 6.32, the Product is 553.632, the ſame as before, which is the Area of the given Hexagon.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 12.64, that Extent will reach from 43.8, the ſame Way to 553.632: Or, extend from 2 to 12.64, that Extent will reach from 87.6 to 553.632.

Demonſtration. Every regular Polygon is equal to the Parallelogram, or long Square, whoſe Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of ſix equilateral Triangles: And the Parallelogram OPQR is alſo compoſed of ſix equilateral Triangles, that is, five whole ones, and two Halves; therefore the Parallelogram is equal to the Hexagon.

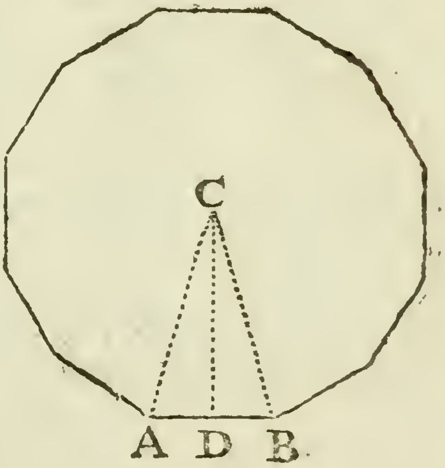
A TABLE *for the more ready finding the Area of a Polygon.*

Number of Sides.	Names.	Multipliers.
3	Trigon	·433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633912
8	Octagon	4.828427
9	Enneagon	6.181824
10	Decagon	7.694209
11	Endecagon	9.365641
12	Dodecagon	11.196152

Multiply the Square of the Side by the Tabular Number, and the Product is the Area of the Polygon.

How to find theſe Tabular Numbers.

Theſe Numbers are found by Trigonometry, thus: Find the Angle at the Center of the Polygon by dividing 360 Degrees by the Number of Sides of the Polygon.



Example. Suppoſe each Side of the Dodecagon annexed be 1, and the Area be required.

Divide 360 by 12 (the Number of Sides), and the Quotient is 30 Degrees for the Angle ACB; the Half of which is 15, the Angle DCB, whoſe Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then ſay:

As s, DCB 15 Degrees is	Co-ar.	0.587004
to .5 the Half-ſide DB. Log.		9.698970
ſo is s, CBD 75 Degrees,		9.984944
to the Perpendicular CD 1.866025		0.270918

Then 1.866025 multiplied by 6 (the Half-perimeter) the Product is 11.196152 the Area of the Dodecagon required.

§ IX. *Of a* Circle.

A Circle is a plain Figure, contained under one Line, which is called a Circumference, unto which all Lines drawn from a Point in the Middle of the Figure, called the Center, and falling upon the Circumference, are equal the one to the other. The Circle contains more Space than any plain Figure of equal Compaſs.

Problem 1. Having the Diameter and Circumference, to find the Area.

The Rule.

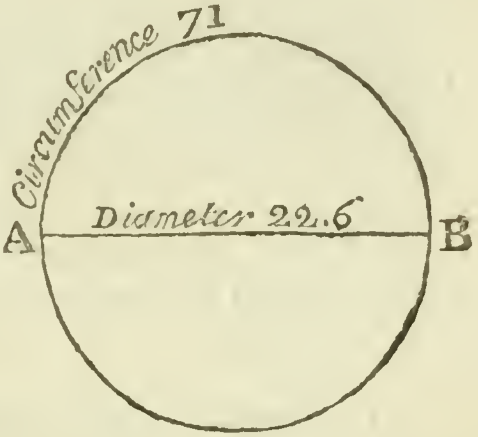
Every Circle is equal to a Parallelogram, whoſe Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply

tiply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Circumf.
11.3 Half Diameter.

1065
355
355

401.15



Thus, if the Diameter of a Circle (that is, the Line drawn croſs the Circle through the Center) be 22.6; and if the Circumference be 71, the Half of 71 is 35.5, and the Half of 22.6 is 11.3; which multiplied together, the Product is 401.15, which is the Area of the Circle.

Demonſtration. Every Circle may be conceived to be a Polygon of an infinite Number of Sides, and the Semidiameter muſt be equal to the Perpendicular of ſuch a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiplied by half the Diameter, gives the Area as aforeſaid.

Or (with *F. Ignat. Gaſton Pardies*) "Every Circle "is equal to a Rectangular Triangle, one of whoſe "Legs is the Radius, and the other a Right Line "equal to the Circumference of the Circle: For "ſuch a Triangle will be greater than any Polygon "inſcribed in, and leſs than any Polygon circum- "ſcribing the Circle, by the 24th, 25th, 26th, and "27th Articles of the fourth Book of his Elements "of Geometry; and therefore muſt be equal to the "Circle.

"For

" For (ſays he) ſhould it be greater than the " Circle, be the Exceſs as little as it will, a Polygon " may be circumſcribed about the Circle, whoſe " Difference, from the Circle, ſhall be yet leſs than " the Difference between the Circle and the rectan- " gular Triangle; and that that Polygon will be " leſs than the Triangle, is abſurd; and if it be " ſaid, that this rectangular Triangle is leſs than " the Circle, an inſcribed Polygon may be made, " which ſhall be greater than that Triangle; which " is impoſſible.

" This cannot but be admitted as a Principle, That " if two determinate Quantities, A and B, are ſuch " that if every imaginable Quantity, which is greater " or leſs than A, is alſo greater or leſs than B, theſe " two Quantities A and B muſt be equal.

" And this Principle being granted, which is in " a manner ſelf-evident, it may directly be proved, " that the Triangle (before-mentioned) is equal to " the Circle; becauſe every imaginable inſcribed " Figure, which is leſs than the Circle, is alſo leſs " than the Triangle; and every circumſcribed " Figure, greater than the Circle, is alſo greater " than the Triangle."

Problem 2. Having the Diameter of a Circle, to find the Circumference.

As 7 to 22, ſo is the Diameter to the Circumference.

Or, as 113 to 355, ſo is the Diameter to the Circumference.

Or, as 1 to 3.141593, ſo is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiplied by 22, and the Product is 497.2; which, divided by 7, gives 71.028 for the Circumference.

ference. Or (by the ſecond Proportion) if 22.6 be multiplied by 355, the Product will be 8023; this divided by 113, the Quotient is 71, the Circumference. Or (by the third Proportion) if 22.6 be multiplied into 3.141593, the Product is 71.0000018 the Circumference; which two laſt Proportions are the moſt exact.

```
  22.6                 355
    22                22.6
  ----               -----
   452                2130
  452                 710
  -----              710
7)497.2(71.028     -------
                  113)8023.0(71.
                       791
                       ---
                       113
                       113
                       ---
                       ...
 3.141593
     22.6
 ---------
 18849558
 6283186
6283186
----------
71.0000018
```

By Scale and Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle to the Circumference was never yet exactly found, notwithſtanding many eminent and learned Men have laboured very far therein; among which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the ſaid Proportion to 36 Places of Decimals, which are engraven upon his Tombſtone in St. *Peter*'s Church in *Leyden*; which Numbers are theſe:

Diameter.

Diameter.

1.00000.00000.00000.00000.00000.00000.00000.

Circumference.

3.14159.26535.89793.23846.26433 83279.50288.

Of which large Number, theſe ſix Places, 3.14159, anſwering to the Diameter 1.00000, may be ſufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe: But in the following Work I ſhall uſe ſometimes one of them, and ſometimes another; but for the moſt Part that of *Van Culen*, as being moſt exact.

Problem 3. Having the Circumference of a Circle, to find the Diameter.

As 1 is to .318309, ſo is the Circumference to the Diameter.

Or, as 355 to 113, ſo is the Circumference to the Diameter.

Or, as 22 to 7, ſo is the Circumference to the Diameter.

Let the Circumference be 71 (as in the former Circle), if .318309 be multiplied by 71 (as by the firſt Proportion), the Product will be 22.5999239 for the Diameter. Or, by the ſecond Proportion, 113 multiplied by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiplied by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309

.318309	113	71
71	71	7
318309	113	22)497(22.59
2228163	791	44
22.599939	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
	...	2

Thus, by both the firft Proportions, the Diameter is 22.6, but by the laft it falls fomething fhort.

By Scale and Compaffes.

Extend the Compaffes from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter fought.

Or, you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the fame will reach from 71 to 22.6, as before.

Note, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters (by *Euclid.* 12.2). Now, the Area of a Circle, whofe Diameter is 1, will be .785398, according to *Van Culen*'s Proportion before-

before-mentioned; but for Practice .7854 will be fufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, fo is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.15 (the Area of the given Circle): But,

According to *Metius*'s Proportion,

As 452 : 355 :: 510.76 : 401.15, the fame as before.

But, if you ufe *Archimedes*'s Proportion, fay,

As 14 : 11 :: 510.76 : 401.31; which Area is greater than by the two former Proportions; though in fmall Circles this is near enough the Truth. See the Working of all thefe.

```
     22.6 Diameter of the former Circle.
     22.6
     ----
     1356
     452
    452
   ------
   510.76 the Square of the faid Diameter.

As 1 : .7854 :: 51076
        .7854
       ------
       204304
      255380
     408608
    357532
   ----------
   401.150904 the Area.
```

By

By Scale and Compaſſes.

The Extent from 1 to 22.6, being twice turned over from .7854, will fall at the laſt upon 401.15, the Area.

```
113            As 452 : 355 : : 510.76
  4                                355
---                             ------
452                             255380
                               255380
                              153228
                           -----------
                       452)181319.80(401.15
                          1808
                          ----
                           519
                           452
                          ----
                            678
                            452
                           ----
                           2260
                           2260
                           ----
                           ....
```

As 14 : 11 : : 510.76

```
          510.76
              11
        --------
14)5618.36(401.31
   56
   --
    18
    14
    --
     43
     42
     --
      16
      14
      --
       2
```

Problem 5. Having the Circumference of a Circle to find the Area.

Becauſe the Diameters of Circles are proportional to their Circumferences; that is, as the Diameter of one Circle is to its Circumference; ſo is the Diameter of another Circle to its Circumference: Therefore the Areas of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

Sq. Cir. Area. Sq. Circumf.
As 1 : .07958 : : 5041

```
              5041
          --------
              7958
22           31832
 4          397900
--        --------
88       401.16278
```

Or

Or thus:

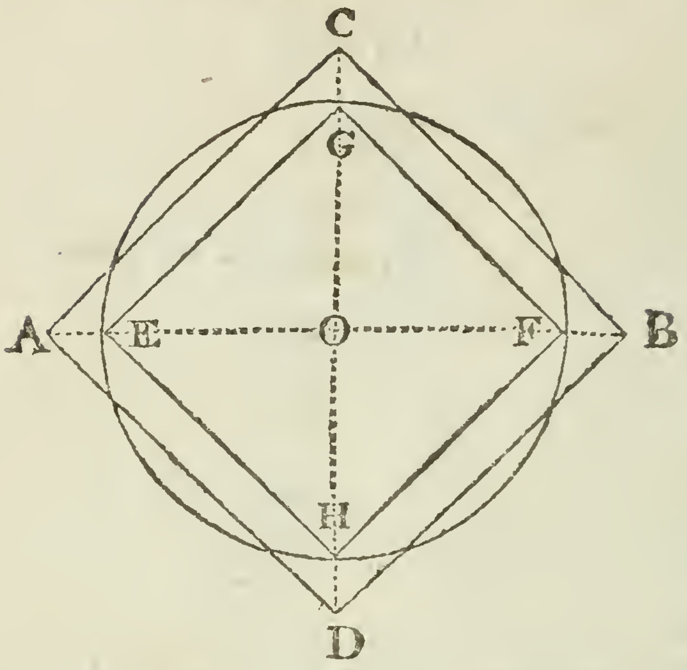
```
As 88 : 7 : : 5041
                 7
              ----
          88)35287(400.98 Area.
             352
             ----
              870
              792
              ----
355           780
  4           704
---           ----
1420           76
```

Or, As 1420 : 113 : : 5041 : 401.15 Area.

Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

```
As 1 : .8862 : : 22.6 (the Diameter)
        22.6
      ------
       53172
      17724
     17724
     -------
To 20.02812 the Side of the Square AC.
```



Let the Diameter of the Circle EF or GH, be 22.6 (as before), to find the Side of the Square AC, AD, *&c.* If .8862 be multiplied by 22.6, the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiplied Square-wife, that is, by itfelf, it will produce 401.1255907344, which is nearly equal to the Area found in the laft Problem.

You may find the Side of the Square equal, by extracting the Square Root out of the Area of the given Circle.

```
   . . .
  401.15(20.0287295 Side of a Square.
  4
 ------
4002)01.1500
         8004
       ------
40048)349600
 ....320384
       ------
        29216
        28034
       ------
         1182
          801
       ------
          381
          360
         ----
           21
           20
          ---
            1
```

N. B. By this Method of extracting the Square Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

```
As 1 : .2821 : : 71 (the Circumference)
          71
       ------
         2821
        19747
      -------
      20.0291 the Side of the Square.
```

Problem 8. Having the Diameter, to find the Side of a Square, which may be inſcribed in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inſcribed will be .7071. Therefore,

As 1 : .7071 : : 22.6
22.6
___
42426
14142
14142
___
To 15.98046 the Side EG inſcribed.

Or, if you ſquare the Semidiameter and double that Square, the Square Root of the doubled Square will be the Side of the Square inſcribed. For (by *Euclid.* 1.47), the Square of the Hypothenuſe EG is equal to the Sum of the Squares of the other two Legs, EO and OG.

11.3 Semidiameter.
11.3
___
339
113
113
___
127.69 the Square of EO, which double, becauſe EO=OG.
2
___
255.38(15 98 Root, which is the Side of the Square
1
___
25)155
125
___
309)3038
2781
___
3188)25700
25504
___
196

Problem

Problem 9. Having the Circumference, to find the Side of a Square which may be inſcribed.

If the Circumference be 1, the Side of the Square inſcribed will be .2251. Therefore,

$$\begin{array}{r} \text{As } 1 : .2251 :: 71 \\ 71 \\ \hline 2251 \\ 15757 \\ \hline 15.9821 \end{array}$$

15.9821 the Side of the Sq. EG.

Becauſe that in each of the four laſt Problems, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be eaſy to work them with Scale and Compaſſes; for if you extend the Compaſſes from the firſt to the ſecond, that Extent will reach from the third to the fourth. As in the laſt Problem, where the Proportion is as 1 to .2251, ſo is 71 to the Side of the Square 15.9821. Here extend the Compaſſes from 1 to .2251; that Extent will reach from 71 to 15.98; and ſo of the reſt. But the fifth muſt be wrought like the fourth, thus; extend the Compaſſes from 1 to 71; that Extent, turned over the ſame Way from .07958, will fall, at laſt, upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter is 1.2732. Therefore,

```
       Area. Sq. Diam.   Area.
       As 1 : 1.2732 : : 401.15
                 401.15
                 ------
                  63660
                 12732
                12732
              509280
              ---------
              .  .  .  .  .
              510.744180(22.599 the Diameter.
              4
           -----
           42)110
               84
           -----
           445)2674
               2225
               ----
           4509)44941
                40581
                -----
          45189)436080
                406701
                ------
                 29379
```

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.2732; that Extent will reach from 401.15 to 510 74, &c. Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.15 and .7854, into two equal Parts, and one of thoſe Parts will reach from 1 to 22.6, the Diameter ſought.

Problem

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

```
      Ar. Sq. Circumf.     Area.
      As 1 : 12.56637 : : 401.15
                401.15
             ---------
               6283185
              1256637
             1256637
            50265480
           ----------
             . . . . . .
            5040.99932550(70.9999 Root.
            49
           ------
      1409)14099
            12681
           ------
     14189)141893
            127701
            -------
    141989)1419225
            1277901
            --------
   1419989)14132450
            12779901
            ---------
             1352549
```

By Scale and Compaffes.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of thofe Parts will reach from 1 to 71, the Circumference fought.

Problem 12. Having the Area, to find the Side of a Square infcribed.

If the Area of a Circle be 1, the Area of a Square infcribed within that Circle will be .6366. Therefore,

```
As 1 : 401.15 : : .6366
        .6366
      -------
       240690
      240690
     120345
    240690
   ----------
    . . . . . .
    255.372090(15.980 Root, which is the Side
    1                      [of the Square fought.
    ---
 25)155
    125
    -----
309)3037
     2781
    -----
3188)25620
      25504
     ------
      11690
```

The fame Reafon may be given for the laft Proportion, that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but alfo all Figures infcribed or circumfcribed, have the Squares of their like Sides proportioned to the Squares they are infcribed in, or circumfcribed about; and alfo to the Figures themfelves: The Square of any Side of one Figure is

is to the Area of that Figure, as the Square of the like Side of another ſimilar Figure is to the Area thereof; as you may find proved at large in *Euclid*, *Sturmius*, *Matheſis*, *Enucleata*, and other Authors; but will be too large to inſert in this Place.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumſcribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumſcribe that Square, will be 1.4142. Therefore,

```
As 1 : 1.4142 : : 15.98
         15.98
       -------
        113136
       127278
      70710
     14142
     ---------
     22.598916 the Diameter ſought.
```

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter ſought.

Problem

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal to it.

If the Side of a Square be 1, the Diameter of a Circle equal to it will be 1.128. Therefore,

```
   Side    Diam.  Side of a Square.
As 1  :  1.128 : : 20 0291
                    1.128
                 ---------
                  1602328
                  400582
                 200291
                200291
               ----------
               22.5928248 Diameter.
```

By Scale and Compaffes.

Extend the Compaffes from 1 to 1.128; that Extent will reach from 20.0291 (the Side of the Square given) to 22.6, the Diameter of the Circle fought.

Problem 15. Having the Side of a Square, to find the Circumference of a circumfcribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompafs that Square will be 4.443. Therefore,

```
Side Sq.  Circum.  Side Sq.
   As 1 : 4.443 : : 15.98
          15.98
        -------
          35544
         39987
        22215
        4443
        --------
        70.99914 the Circumference.
```

By Scale and Compaffes.

Extend the Compaffes from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal to it.

If the Side of a Square be 1, the Circumference of a Circle that will be equal to it is 3.545. Then,

As 1 : 3.545 : : 20.0291

$$\begin{array}{r} 20.0291 \\ 3.545 \\ \hline 1001455 \\ 801164 \\ 1001455 \\ 600873 \\ \hline 71.0031595 \end{array}$$

71.0031595 the Circumference.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference ſought.

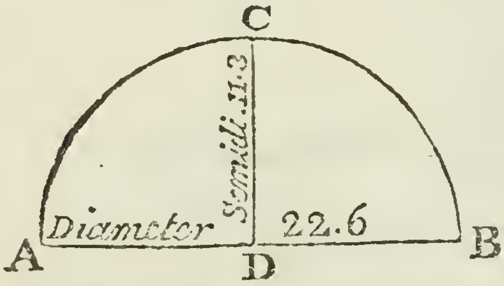
In ſeveral of the foregoing Problems, where the Diameter and Circumference are required, the Anſwers are not exactly the ſame as the Diameter and Circumference of the given Circle, but are ſometimes too much, and ſometimes too little, as in the two laſt Problems, where the Anſwers in each ſhould be 71, the one being too much, and the other too little. The Reaſon of this is, the ſmall Defect that happens to be in the Decimal Fractions, they being ſometimes too great, and ſometimes too little; yet the Defect is ſo ſmall, that it is needleſs to calculate them to more Exactneſs.

§ X. *Of a* SEMICIRCLE.

TO find the Area of a Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch Line) by the Semidiameter, the Product is the Area.



Let ABC be a Semicircle, whoſe Diameter is 22.6, and the half Circumference, or Arch Line, ABC, is 35.5, the Half of it is 17.75, which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

17 75	the half Arch Line.
11.3	the Semidiameter.
5325	
1775	
1775	
200 575	the Area of the Semicircle.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may ſay, by the *Rule of Three,*

As 1 is to .3927, ſo is the Square of the Diameter to the Area.

By

By Scale and Compaſſes.

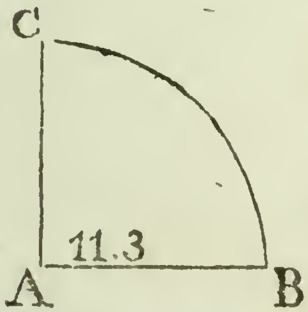
Extend the Compaſſes from 1 to the Diameter 22.6; that Extent turned twice from .3927, will reach, at the laſt, to 200.575.

§ XI. *Of a* QUADRANT.

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch Line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle), by the Semidiameter, and the Product is the Area of the Quadrant.



Let ABC be a Quadrant, or fourth Part of a Circle, whoſe Radius, or Semidiameter, is 11.3, and the half Arch Line 8.875; theſe multiplied together, the Product is 100.2875 for the Area.

Theſe are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and a fourth Part for the Quadrant.

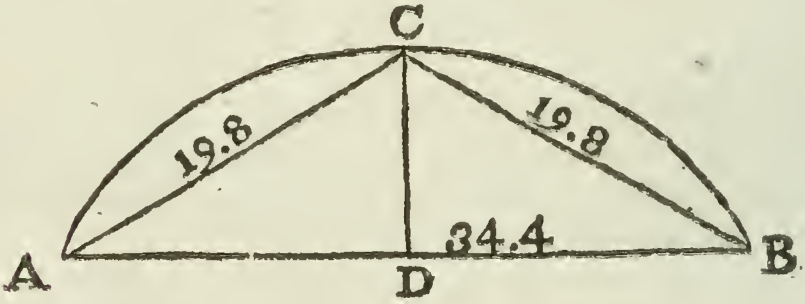
Before I proceed to ſhew how to find the Area of the Sector, and Segment of a Circle, I ſhall ſhew how to find the Length of the Arch Line, ſeveral Ways.

To find the Length of the Arch Line.

Multiply, continually, the Radius, the Number of Degrees in the given Arch, and the Number * .01745329; the Product will be the Length of the Arch-line ACB.

Another Way.

Multiply the Chord of half the Segment AC or CB by 8, and from the Product ſubtract the Chord of the whole Segment AB, and divide the Remainder by 3; the Quotient is the Arch Line ACB ſought, nearly.



19.8	AC
8	
158.4	
34.4	AB
3)124	
Arch Line 41.333	

* When the Radius is 1 the Semi-circumference of the Circle is 3.14159265, &c. and this Number divided by 180, the Degrees in a Semicircle, gives .01745329 for the Length of one Degree when the Radius is Unity.

Another

Another Way.

From the double Chord of half the Segment's Arch, fubtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch Line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which, if you fubtract 34.4, the Remainder is 5.2, which, divided by 3, the Quotient is 1.733; this added to 39.6 (the double Chord of the half Segment), the Sum is 41.333. So if the Arch Line ACB was ftretched out ftrait, it would then contain 41.333 fuch Parts as the Chord AB contains 34.4 of the like Parts.

The two laft Rules may eafily be proved out of the Table of natural Sines; thus,

Suppofe (in the former Figure) the Arch ACB to contain 120 Degrees; the natural Sine of half, *viz.* of 60 Degrees, is 86602; which, being doubled, is 173204, which is the Chord of the whole 120 Degrees, that is, AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the fecond Rule, multiply 100000 by 8, the Product is 800000; from which fubtract 173204 (the Chord AB), and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch Line ACB, according to the fecond Rule. Now let us examine how near this comes to the true Quantity of the Arch propofed. If the Radius or Semidiameter of a Circle be 100000 (as in the Table of Sines), then the Circumference will be 628318; and becaufe 120 Degrees is the third Part of the Circle, take the third Part of 628318, which is 209439, which is the true Quan-

tity of the Arch ACB in ſuch Parts as the Semidiameter contains 100000, and differs from that before found 507, which is a thing inconſiderable in *Practical Menſuration.* The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the ſame as above; either of the Rules gives the Quantity of the Arch Line too little, and the greater the Arch, the greater the Error. But if you know the Degrees that are contained in the Segment's Arch, and would have the Arch Line very exactly, it will be beſt to uſe the firſt Rule; an Example of which follows.

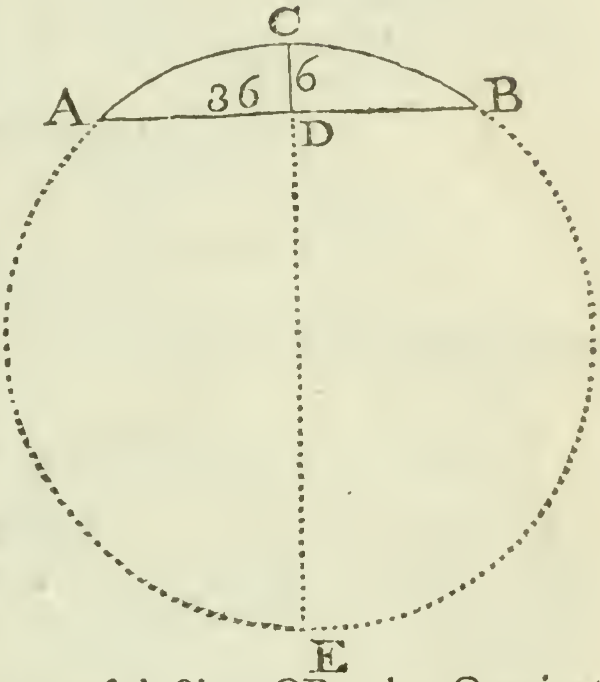
Suppoſe the Diameter of a Circle be 22.6, and the Arch to contain 52 Degrees 15 Minutes (the Decimal of 15 Minutes is .25); then

$$\begin{array}{r}
\overset{\circ}{52.25} \\
11.3 \\
\hline
15675 \\
5225 \\
5225 \\
\hline
590.425 \\
.01745 \\
\hline
2952125 \\
2361700 \\
4132975 \\
590425 \\
\hline
10.30291625
\end{array}$$

So the 52 Degrees 15 Minutes will contain 10.309 of ſuch Parts as the Diameter contains 22.6, or as the Circumference contains 71.

Thus have I ſhewn ſeveral Ways of finding the Meaſure of the Curve Line of any Part of a Circle very near the Truth. The next Thing I ſhall ſhew, is,

How to find the Diameter of a Circle by having the Chord and verſed Sine of the Segment, Arithmetically.



Becauſe the Chord AB cuts the Diameter EC at right Angles, therefore the Semichord AD, or DB, is a mean proportional Line between the Parts of the Diameter CD and DE (by *Euclid* 6. 13.); therefore if you ſquare the Semichord AD, or DB, and divide the Square by the verſed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given verſed Sine CD, and the Sum is the Diameter ſought.

Example. Let ACB be a Segment given, whoſe Chord AB is 36, and the verſed Sine CD 6; half 36 is 18, which, ſquared, makes 324; this divided by 6, the Quotient is 54: To which add 6, the Sum is 60, the Diameter of the Circle CE.

18 half the Chord.
18
———
144
18
———
6)324 the Square of AD.
———
54 the Part wanting DE.
6 the versed Sine CD, add.
———
60 the Diameter CE.

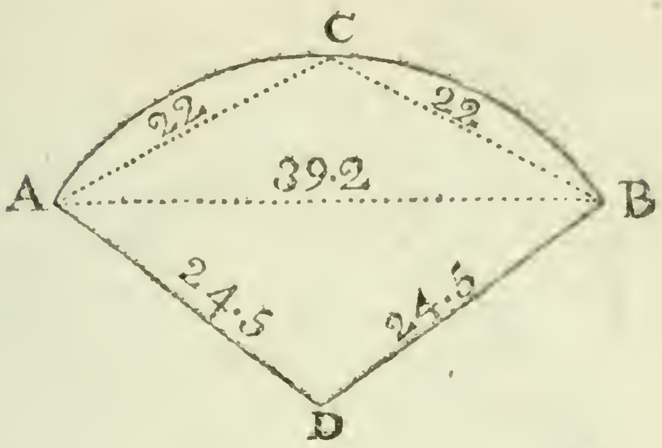
§ XII. *Of the Sector of a* Circle.

A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are supposed not to make one Right Line, and a Part of the Circumference: Whence a Sector may be either less or greater than a Semicircle. To find the Area or superficial Content thereof, this is

The RULE.

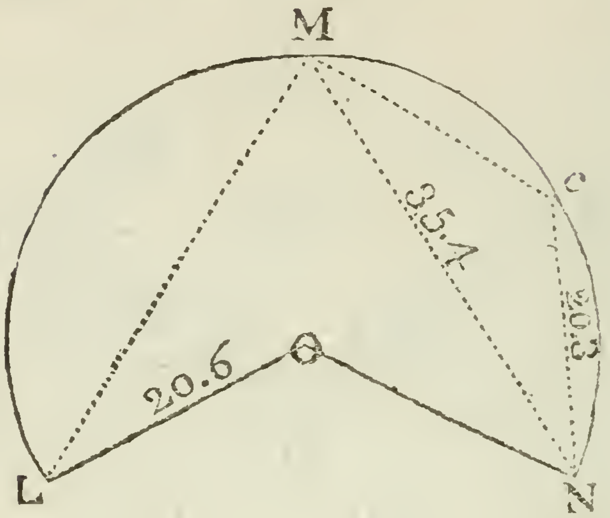
Multiply half the Arch Line by the Semidiameter, and the Product is the Area.

Let ADBC be the Sector of a Circle given, whose Semidiameter AD or BD is 24.5, and the Arch Line ACB (by the second Rule, *Pag.* 112) I find to be 45.6; the half thereof, 22.8, being multiplied by 24.5 (the Semidiameter) the Product is 558.6; which is the Area of the Sector ACBD.




```
      22
       8
    ----
     176
    39.2 Subtrah.
   -----
  3)136.8
   -----
    45.6 Arch Line.
    ----
    22.8 Half.
    24.5 Semidiam.
    ----
    1140
    912
   456
  ------
  558.60 the Area.
```

Again: Let LMNO be a Sector greater than a Semicircle, whose Semidiameter LO or NO is 20.6, and by the Rule, *Pag.* 112, half the Arch, or M *c* N, is found to be 42.333; which, multiplied by 20.6, the Semidiameter, makes 872.0598 for the Area of the Sector LMNO. See the following Work.



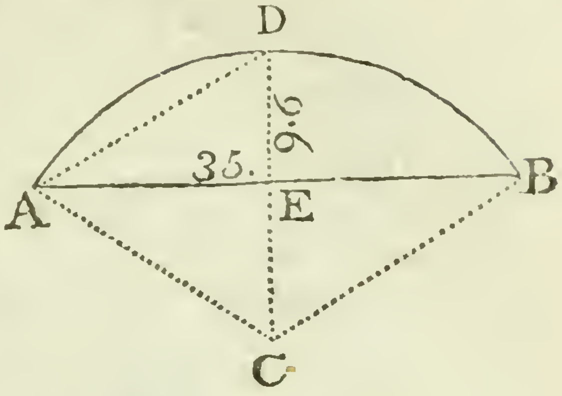
20.3	Chord N *c*.	42.33	
8		20.6	
162.4		253998	
35.4	Chord MN.	846660	
3)127.0		872.0598	Area.
42.333	Arch Line, M *c* N.		

§ XIII. *Of the Segment of a* Circle.

A Segment of a Circle is a Part terminated by a Right Line leſs than the Diameter, called a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you muſt, firſt, find the Center of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the laſt Section), find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and ſubtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Other-



Otherwiſe you may, without deſcribing the Figure, find the Semidiameter of the Circle by the Rule (*Pag.* 115.) and by the Rule (*Pag.* 112.) find the Arch Line; then multiply half the Arch Line by the Semidiameter; ſo have you the Area of the Sector: Then ſubtract the verſed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle. Then ſubtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment. See the Work.

```
      2)35=AB
      -------
        17.5
        17.5
       -----
         875
      1225
       175
     ------
9.6)306.25(31.9
   ------   9.6  add.
      182  -----
       865  41.5  the Diameter of the Circle.
      ----  ----
       1    20.75  the Semidiameter.
             9.6   DE Subtract.
           -----
            11.15  remains the Perpendicular EC.
```

11.15 the Perpendicular EC.
17.5 half the Chord AE or EB.

5575
7805
1115

195.125 the Area of the Triangle.

306.25 the Square of AE.
92.16 the Square of DE the verfed Sine.

398.41 Sum.

The Square Root of which is 19.96, the Chord AD.
8

159.68
Sub. 35 the Chord AB.

3)124.68

2)41.56 the Arch Line.

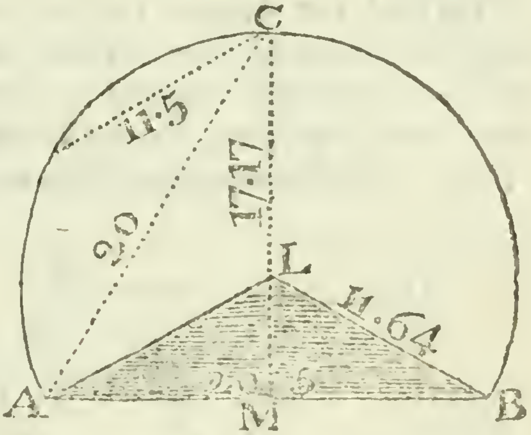
20.78 half.
20.75 Semidiameter.

10390
14546
41560

From	431.1850	Area of the Sect.
Subtract	195.125	Area of the Tri.
Remains	236.06	Area of the Seg.

Again:

Again: Let MACBM be a Segment greater than a Semicircle; obferve the former ru'es in all refpects, as in the laft Example; only, inftead of fubtracting the Area of the Triangle out of the Area of the Sector, here you muft add them together, as may plainly appear by the following Figure.



11.5
8

92.0
20

3)72

24 HalfArchLine

11.64 Semidiam.
24

4656
2328

279.36 Area of the Sector LACBL.

17.17
11.64

5.53 LM.

10.25 half the Bafe MA.
5.53 the Perpendicular LM.

3075
5125
5125

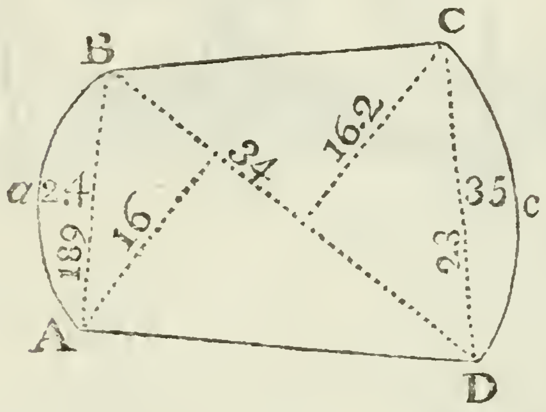
56.6825 the Area of the Triangle ALM.
279.36 the Area of the Sector add.

336.0425 the Area of the Segment fought.

§ XIV. *Of Compound* Figures.

MIXED or compound Figures are ſuch as are composed of rectilineal and curvilineal Figures together.

To find the Area of ſuch mixed Figures, you muſt find the Area of the ſeveral Figures of which the whole compound Figure is compoſed, and add all the Areas together, and the Sum will be the Area of the whole compound Figure.



16.2	add.
16	
32.2	Sum.
16.1	half.
34	Diagonal.
644	
483	
547.4	Area of the Trapezium.

Half the Arch Line A a B.	19.8047
Semidiameter of the Arch AB.	9.85
	990235
	1584376
	1782423
Area of the Sector	195.076295

From

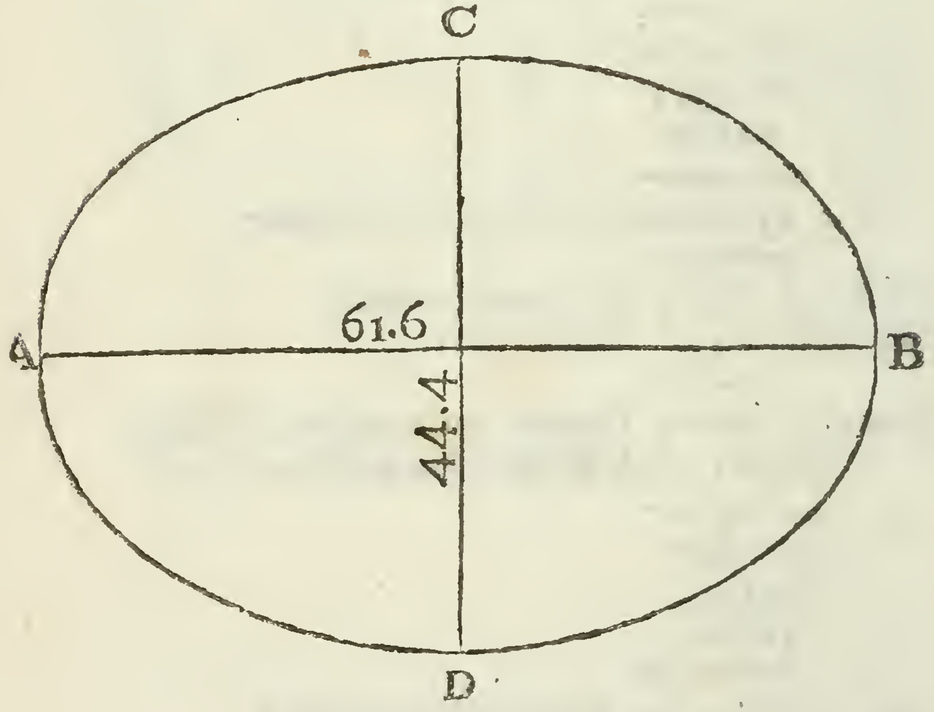
From	19.8047	Semidiameter.
Subtract	2.4	Verfed Sine.
	17.4047	Perpend. of the Triangle.
	9.45	Half the Chord AB.
	870235	
	696188	
	1566423	
	164.474415	the Area of the Triangle.
	195.076295	the Area of the Sector.
	30.60188	the Area of the Segment A a B A.
	12.19	half the Arch Line C c D.
	20.64	Semidiameter.
	4876	
	7314	
	24380	
	251.6016	the Area of the Sector.
From	20.64	the Semidiameter.
Subtract	3.5	Verfed Sine.
Rema.	17.14	Perpendicular of the Triangle.
	11 5	half the Chord DC.
	8570	
	1714	
	1714	
Sub.	197.110	Area of the Triangle.
From	251.602	Area of the Sector.
Rem.	54.492	the Area of the Segment C c DC.
	30.602	the Area of the Segment A a BA.
	547.4	the Area of the Trapezium.
Sum	632.494	the Area of the Whole.

§ XV. *Of an* ELLIPSIS.

AN Ellipſis, or Oval, is a Figure bounded by a regular Curve Line, returning into itſelf; but of its two Diameters, cutting each other in the Center, one is longer than the other, in which it differs from the Circle. To find the Area of it, this is

The RULE.

Multiply the tranſverſe Diameter by the Conjugate, and multiply that Product by .7854, this laſt Product is the Area of the Oval.



61.6 the tranſverſe Diameter.
44.4 the conjugate Diameter.

2464
2464
2464

2735.04 the Rectangle.
.7854 the Area of Unity.

1094016
1367520
2188032
1914528

2148.100416 the Area of the Oval.

Demonſtration. If you circumſcribe any Ellipſis with a Circle, and ſuppoſe an infinite Number of Chord Lines drawn therein, all parallel to the conjugate Diameter, as theſe in the following Figure; then it will be,

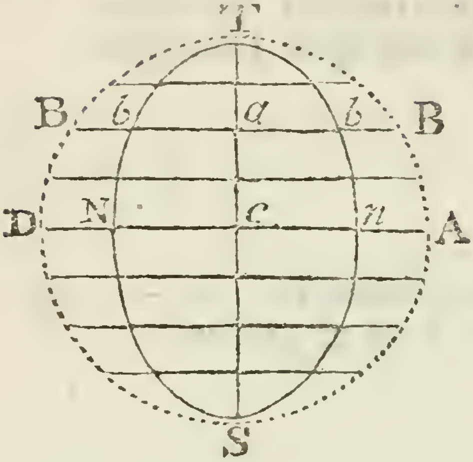
As DA, the Diameter of the Circle, is to N n, the conjugate Diameter of the Ellipſis; ſo is B a B, any Chord in the Circle, to b a b, its reſpective Ordinate in the Ellipſis.

For, according to the Property of the Circle,

it is	1	$aS \times Ta = \square Ba$.
And		by the Property of the Ellipſis,
it is	2	$\square TC : \square NC :: aS \times Ta : \square ba$,
1, 2	3	$\square TC : \square NC :: \square Ba : \square ba$.
3, hence	4	$TC : NC :: Ba : ba$.
Conſq.	5	$2\,TC : 2\,NC :: 2\,Ba : 2\,ba$.
That is	6	$DA : Nn :: BaB : bab$.

But the Sum of an infinite Series of ſuch Chords as B a B, do conſtitute the Area of the Circle. And the Sum of the like Series of their reſpective Ordinates, as b a b, do conſtitute the Area of the Ellipſis:

 There-



Therefore T S : N n : : Circle's Area : the Ellipfis Area. But T S : N n : : □ TS : TS × N n; whence it follows, that,

□ T S : Circle's Area : : T S × N n : Ellipfis Area.

Confequently, as 1 is to .7854, fo is the Rectangle, or Product, of the tranfverfe and conjugate Diameter of any Ellipfis to its Area.

Hence it is eafy to conceive, that the Square Root of the Product of the tranfverfe and conjugate Diameters, will be the Diameter of a Circle equal to the Ellipfis.

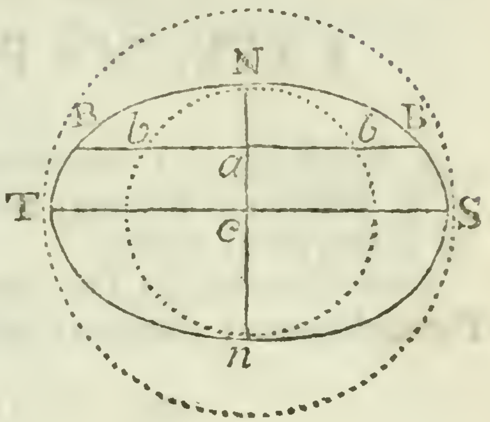
Hence alfo the Segments of an Ellipfis, and its circumfcribing Circle (whofe Bafes are parallel to the conjugate Diameter, and of the fame Height), are in Proportion one to another as their Bafes are. That is,

B a B : b a b : : Area Segment BTB : Area Segment b T b.

Or, TS : N n : : Area Segment BTB : Area Segment b T b

The Area of every Ellipfis is a mean Proportional between the Area of its circumfcribing and infcribed Circles.

The



The Truth of this may eaſily be deduced from the laſt; for 'tis already proved, that □ TS : TS × Nn : : circumſcribing Circle's Area : Ellipſis Area.

But □ TS : TS × Nn : : TS × Nn : □ Nn. Therefore Ellipſis Area: inſcribed Circle's Area : TS × Nn : □ Nn.

Example. Let TS = 36, and Nn = 18.4.
Then □ TS = 1296, and □ Nn = 338.56.
Then 1296 × .7854 = 1017.8784 great Circle's Area;
And 338.56 × .7854 = 265.905, &c. leſſer Circle's Area;
And 36 × 18.4 = 662.4 × .7854 = 520.24896, which is the Area of the Ellipſis; then it will be,

1017.878 : 520.24896 : : 520.24896 : 265.905024.

That is, As the greater Circle's Area is to the Area of the Ellipſis, ſo is the Area of the Ellipſis to the Area of the leſſer Circle.

From hence it follows, that all Segments of an Ellipſis, and its inſcribed Circles (whoſe Baſes are parallel to the tranſverſe Diameter, and have the ſame Height) are in Proportion one to another as the Area of the Ellipſis and Circle are.

That is, as the Area of the Circle is to the Area of the Ellipſis, ſo is the Segment b N b : to the Segment BNB;

Or, Nn : TS : : Area Segment b N b : Area Segment BNB.

§ XVI.

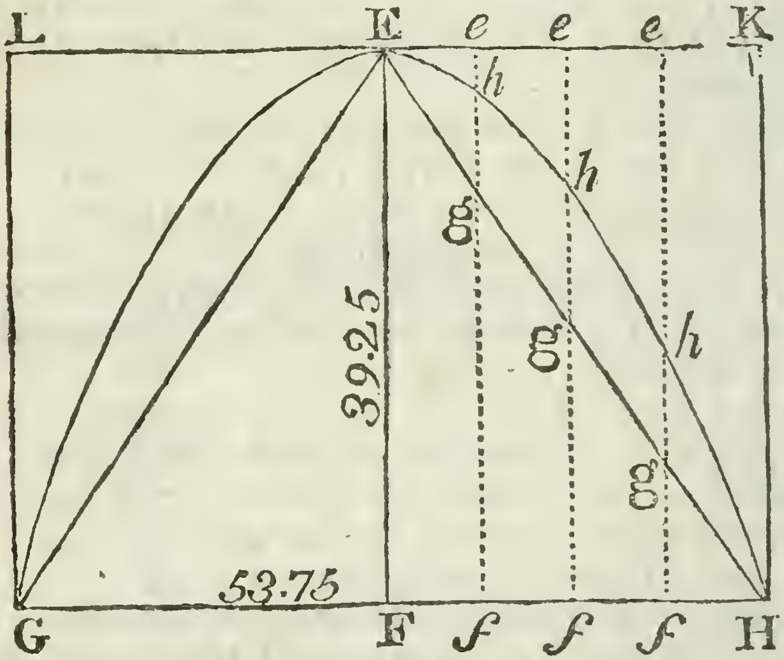
§ XVI. *Of a* PARABOLA.

A Parabola is a curvilineal Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is Two-thirds of its circumſcribing Parallelogram; therefore to find the Area, this is

The RULE.

Multiply the Baſe, or greateſt Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the laſt Product by 3, the Quotient will be the Area of the Parabola.



$$\begin{array}{rl}
53.75 & \text{the Ordinate GH.} \\
39.25 & \text{the Perpendicular EF.} \\
\hline
26875 & \\
10750 & \\
48375 & \\
16125 & \\
\hline
2109.6875 & \\
2 & \\
\hline
3)4219.3750 & \\
\hline
1406.4583 & \text{the Area.}
\end{array}$$

Demonſtration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, *&c.* and through the Diviſions draw Lines, as e f, e f, *&c.* parallel to the Axis EF. Suppoſe alſo EF to be 4.

Then, I ſay, the Parabolic Space Eh HF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

For, firſt, g f, g f, g f, *&c.* are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, fe : ge : : ge : he; but he, in the Axis EF, $=0$; and in the firſt Parallel ef muſt be equal to $\frac{1}{4}$, in the next e f muſt be equal to $\frac{4}{4}$, in the third to $\frac{9}{4}$, and ſo on, in a duplicate arithmetical Progreſſion.

For e f $(=4)$: ge $(=1)$: : ge $(=1)$: e h $(=\frac{1}{4})$. And the ſecond e f $(=4)$: eg $(=2)$: : eg $(=2)$: e h $(=\frac{4}{4})$, *&c.* And thus it will be, if the Lines F f, ff, *&c.* be again biſected, *&c. ad infinitum*, ſo that all the Indiviſibles of the trilinear Space EK H h E will be in a duplicate arithmetical Progreſſion increaſing. But the Sum of a Rank of ſuch Terms is ſubtriple to a Rank of as many equal to the greateſt (by Lemma 3); wherefore the whole trilinear Space EK H h E is to the Parallelogram as 1 to 3; and, conſequently, the remaining parabolic Space muſt be to it as 2 to 3; which was to be proved.

And

And fince the Triangle FEH is to the Parallelogram as 1 to 2, it muft be to the Parabola as $1\frac{1}{2}$ to 2, or as 3 to 4; which was to be proved.

Before I proceed to the Menfuration of folid Bodies, I will lay down fuch Lemmas as will be neceffary to facilitate the Demonftration of all fuch Solids.

LEMMA I.

In any Series of equal Numbers (reprefenting Lines or other Quantities, as 1, 1, 1, 1, *&c.* or 2, 2, 2, 2, *&c.* or 3, 3, 3, 3, *&c.* if one of the Terms be multiplied into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers, in arithmetical Progreffion, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, *&c.* (reprefenting a Series of Lines or Roots beginning with a Point) if the laft Term be multiplied into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the laft Term, N = the Number of Terms, and S = the Sum of all the Series; then will NL = 2 S; confequently, $\frac{1}{2}$ NL = S; *viz.* One half of fo many times the greateft Term as there are Terms in the Series.

$$\text{Thus } \begin{cases} 0+1+2+3+4=10 \text{ the Sum}=\frac{1}{2}NL. \\ \overline{4+4+4+4+4=20=NL.} \end{cases}$$

LEMMA III.

If a Series of Squares, whofe Sides or Roots are in arithmetical Progreffion, beginning with a Cypher, *&c.* be infinitely continued, the laft Term being multiplied into the Number of Terms, will be triple

to

to the Sum of all the Series; *viz.* NLL$=$3S; or $\frac{1}{3}$ NLL$=$S.

That is, the Sum of ſuch a Series will be One-third of the laſt or greateſt Term, ſo many times repeated as there are Terms in the Series.

Inſtances in ſquare Numbers.

$$1 \left\{ \frac{0+1+4}{4+4+4} = \frac{5}{12} = \frac{1}{3} + \frac{1}{12}. \right.$$

$$2 \left\{ \frac{0+1+4+9}{9+9+9+9} = \frac{14}{36} = \frac{7}{18} = \frac{1}{3} + \frac{1}{18}. \right.$$

$$3 \left\{ \frac{0+1+4+9+16}{16+16+16+16+16} = \frac{30}{80} = \frac{3}{8} = \frac{9}{24} = \frac{1}{3} + \frac{1}{24}, \right.$$

(*&c.*

From theſe Inſtances it is evident, that as the Number of Terms in the Series do increaſe, the Fraction or Exceſs above $\frac{1}{3}$ does decreaſe, the ſaid Exceſs always being $\frac{1}{6N-6}$; which, if we ſuppoſe the Series to be infinitely continued, will quite vaniſh, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* (as above) be infinitely continued, the Sum of all the Series will be $\frac{1}{4}$ NLLL$=$S.

That is, One-fourth of the laſt Term ſo many times repeated as there are Terms in the Series.

Inſtances

Inſtances in Cube Numbers.

If 0, 1, 2, 3, 4, 5, *&c.* be the Roots of the Cubes,

1 $\left\{\dfrac{0+1+8+27}{27+27+27+27}=\dfrac{36}{108}=\dfrac{4}{12}=\dfrac{1}{4}+\dfrac{1}{12}.\right.$

2 $\left\{\dfrac{0+1+8+27+64}{64+64+64+64+64}=\dfrac{100}{320}=\dfrac{10}{32}=\dfrac{5}{16}=\dfrac{1}{4}+\dfrac{1}{16}.\right.$

3 $\left\{\dfrac{0+1+8+27+64+125}{125+125+125+125+125+125}=\dfrac{225}{750}=\dfrac{45}{150}\right.$
$(=\dfrac{6}{20}=\dfrac{1}{4}+\dfrac{1}{20}.$

From theſe Examples it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{1}{4}$ decreaſes, the Exceſs being always $\dfrac{1}{4N-4}$; which, if we ſuppoſe the Series to be infinitely continued, will become infinitely ſmall, or nothing.

LEMMA V.

If a Series of Biquadrates, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* as before, be infinitely continued, the Sum of all the Terms in ſuch a Series will be $\frac{1}{5}$ NLLLL.

The Truth of this may be manifeſted by the like Proceſs as in the foregoing Lemmas, and ſo on for higher Powers.

LEMMA VI.

The Sum of an infinite Progreſſion, whoſe greateſt Term is a ſquare Number, the other decreaſing by odd Numbers, *viz.* 1, 3, 4, *&c.* is in ſubſeſquialteran Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Inſtances in ſuch Progreſſions.

$$1\quad \left\{\frac{9+8+5}{9+9+9}=\frac{22}{27}=\frac{2}{3}+\frac{4}{27}.\right.$$

$$2\quad \left\{\frac{16+15+12+7}{16+16+16+16}=\frac{50}{64}=\frac{2}{3}+\frac{11}{96}.\right.$$

$$3\quad \left\{\frac{25+24+21+16+9}{25+25+25+25+25}=\frac{95}{125}=\frac{2}{3}+\frac{7}{75}.\right.$$

$$4\quad \left\{\frac{36+35+32+27+20+11}{36+36+36+36+36+36}=\frac{161}{216}=\frac{2}{3}=\frac{17}{216}.\right.$$

From theſe Examples it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{2}{3}$ decreaſes; and if we ſuppoſe the Series to be infinitely continued, that Exceſs will quite vaniſh, and the Sum of the infinite Series will be $\frac{2}{3}$ of ſo many equal to the greateſt.

CHAP. II.

The Menſuration of SOLIDS.

SOLID Bodies are ſuch as conſiſt of Length, Breadth, and Thickneſs; as Stone, Timber, Globes, Bullets, *&c.*

§ I. *Of a* CUBE.

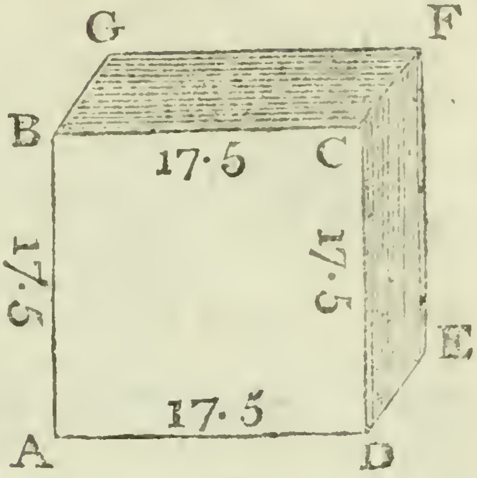
A CUBE is a ſquare Solid, comprehended under ſix geometrical Squares, being in the Form of a Dye. To find the Solid Content, this is

The RULE.

Multiply the Side of the Cube into itſelf, and that Product again by the Side; the laſt Product will be the Solidity, or ſolid Content of the Cube.

```
    17.5
    17.5
    ----
     875
   1225
   175
  ------
  306.25
    17.5
  ------
  153125
 214375
 30625
--------
5359.375
```

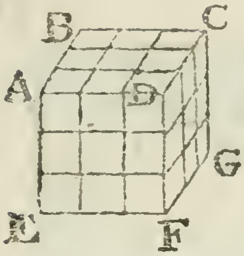
the folid Content of the Cube.



Suppofe ABCDEFG a cubical Piece of Stone or Wood, each Side being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiplied by 17.5, the laft Product is 5359.375, which is 5359 folid Inches and 375 Parts. To reduce the folid Inches to Feet, divide by 1728 (becaufe fo many cubical Inches is a Foot), and the folid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale and Compaffes.

Extend the Compaffes from 1 to 17.5; that Extent turned over twice from 17.5 will reach to 5359, the folid Content in Inches. Then extend the Compaffes from 1728 to 1; that Extent, turned the fame Way from 5359, will reach to 3.1 Feet.



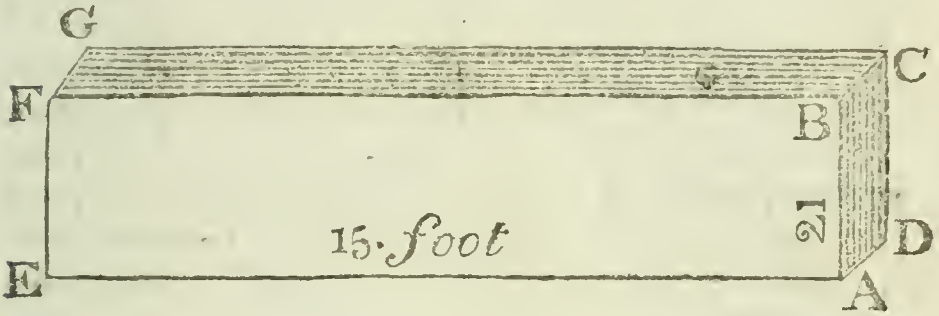
Demonſtration. If the Square ABCD be conceived to be moved down the Plane ADEF, always remaining parallel to itſelf, there will be generated, by ſuch a Motion, a ſolid, having ſix Planes, the two oppoſite ones of which will be equal and parallel to each other; whence it is called a Parallelopipedon, or ſquare Priſm. And if the Plane ADEF be a Square equal to the penetrating Plane ABCD, then will the generated Solid be a Cube. From hence ſuch Solids may be conceived to be conſtituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms. Therefore, if the Area of ABCD be multiplied into the Number of Terms AE, the Product is the Sum of all the Series, (*per* Lemma I.) and, conſequently, the Solidity of the Parallelopipedon or Cube. Or, if the Baſe ABCD, being divided into little ſquare Areas, be multiplied into the Height AE, divided by a like Meaſure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Baſe multiplied by the Number of Diviſions the Side AE contains. Thus, if the Side of the Baſe AB be 3, that multiplied into itſelf is 9, which is the Area of the ſquare Baſe ABCD; then, if AE be likewiſe 3, multiply 9 by 3, and the Product is 27; and ſo many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonſtration it is very plain, that, if you multiply the Area of the Baſe of any Parallelopipedon into its Length or Height, that Product will be the ſolid Content of ſuch a Solid.

§ II.

§ II. *Of a* PARALLELOPIPEDON.

LET ABCDEFG be a Parallelopipedon, or ſquare Priſm, repreſenting a ſquare Piece of Timber or Stone, each Side of its ſquare Baſe ABCD being 21 Inches, and its Length AE 15 Feet.



Firſt, then, multiply 21 by 21, the Product is 441, the Area of the Baſe in Inches; which multiplied by 180, the Length in Inches, and the Product is 79380, the ſolid Content in Inches. Divide the laſt Product by 1728, and the Quotient is 45.9, that is, 45 ſolid Feet and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the ſame as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet 0 Inches 9 Parts; this multiplied again by 15 Feet, gives 45 Feet 11 Inches 3 Parts, that is, 45 Feet and $\frac{11}{12}$ of a Foot and $\frac{1}{4}$ of $\frac{1}{12}$.

See the Work of all theſe.

```
      21
      21
     ---
      21
     42
    ----
     441
     180
   -----
   35280
   441
   -----
1728)79380(45.9
     6912
    -----
    10260
     8640
    -----
    16200
    15552
    -----
      648
```

```
     441
      15
    ----
    2205
    441
    ----
144)6615(45.9
    ----
     855
    1350
    ----
      54
```

```
  F. I.
  1—9
  1—9
  ----
  1—9
  1—3—9
  -------
  3—0—9
  15
  -------
  45—0—0
     7—6
     3—9
  -------
  45—11—3
```

By Scale and Compaffes.

Extend the Compaffes from 12 to 21, and that Extent will reach to near 46 Feet, being twice turned over from 15 Feet; fo the folid Content is almoft 46 Feet.

If the Bafe of the fquared Solid be not an exact Square, but in Form of a rectangle Parallelogram, the Way of meafuring it is much the fame; for, firft, you muft find the Area of the Bafe by multiplying the Breadth by the Depth; and then multiply that Area by the Length of the Piece, as before. Thus,

If

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many ſolid Feet are contained therein?

25	F. I.
9	2—1
	0—9
225	
25	1—6—9
	25
1125	
450	25—0—0
	12—6—0
144)5625(39	1—0—6
432	0—6—3
1305	39—0—9
1296	
9	Anſwer 39 Feet.

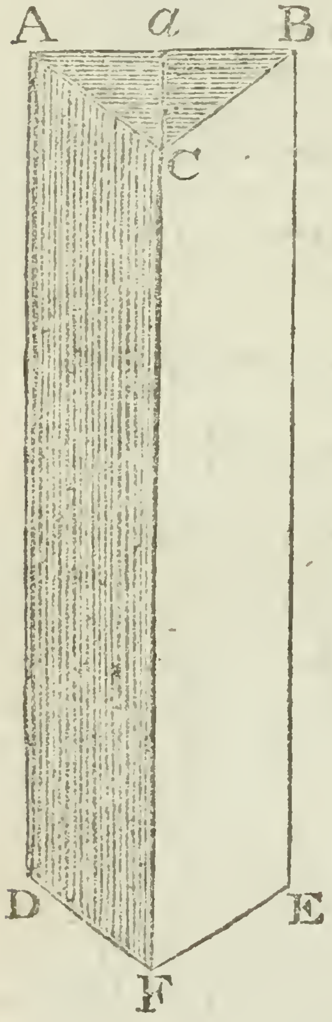
By Scale and Compaſſes.

Firſt, find a mean geometrical Proportion between the Breadth and the Depth; which to do upon the Line of Numbers, you muſt divide the Space upon the Line, between the Breadth and Depth, into two equal Parts; that middle Point will be the mean Proportional ſought: Thus the middle Point between 25 and 9 is at 15; ſo is 15 a mean Proportional between 9 and 25, for 9 : 15 : : 15 : 25; ſo a Piece of Timber of 15 Inches ſquare is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compaſſes from 12 to 15, that Extent, turned twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

§ III.

§ III. *Of a Triangular* PRISM.

A Priſm is a ſolid contained under ſeveral Planes, and having its Baſes like, equal, and parallel. The ſolid Content of a Priſm (whether triangular or multangular) is found by multiplying the Area of the Baſe into the Length or Height, and the Product is the ſolid Content.



Let ABCDEF be a triangular Priſm, each Side of the Baſe being 15.6 Inches, the Perpendicular C *a* 13.51 Inches, and the Length of the Solid 19 5 Feet.

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Baſe; which multiply by the Length 19.5, and the Product is 2054.871; which divide by 144, and the Quotient is 14.27 Feet *ferè*, the ſolid Content.

13.51	144)2054.87(14.27
7.8	144
10808	614
9457	576
105.378	388
19.5	288
526890	1007
948402	1008
105378	
2054.8710	

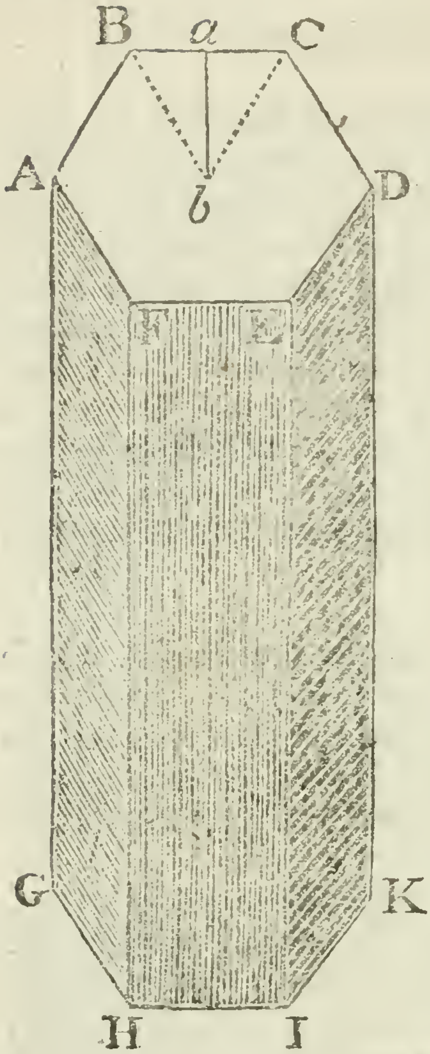
By

By Scale and Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and Half ſide (as before taught), by dividing the Space upon the Line, between 13.51 and 7.8 into two equal Parts; ſo ſhall you find the middle Point between them to be at 10.26, which is the mean Proportional ſought: By this means the triangular Solid is brought to a ſquare one, each Side being 10.26 Inches. Then extend the Compaſſes from 12 to 10.26; that Extent, turned twice downwards from 19.5 Feet, the Length, will at laſt fall upon 14.27, which is 14 Feet and a little above a Quarter.

Let ABCDEFGHIK repreſent a Priſm, whoſe Baſe is a Hexagon, each Side being 16 Inches, the Perpendicular from the Center of the Baſe to the Middle of one of the Sides (a b) 13.84 Inches, and the Length of the Priſm 15 Feet; the ſolid Content is required.

Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Baſe (by § VIII. p. 86), which multiplied by 15 Feet, the Length, the Product is 9964.8; which divided by 144, the Quotient will be 69.2 Feet, the ſolid Content required.



13.84
48

11072
5536

664.32 Ar. of Baſe.
15

332160
66432

144)9964.80(69.2
864

1324
1296

288
288

..

By Scale and Compaſſes.

Firſt, find a mean Proportional between the Perpendicular, and half the Sum of the Sides; that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77. Then extend the Compaſſes from 12 to 25.77; that Extent will reach (being twice turned over) from 15 Feet, the Length, to 69.2 Feet, the Content.

To

To find the ſuperficial Content of any of the forementioned Solids, you muſt take the Girth of the Piece, and multiply by the Length, and to that Product add the two Areas of the Baſes, the Sum will be the whole ſuperficial Content. Example of the hexagonal Priſm laſt mentioned. The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches; which multiplied by 96, the Product is 17280 ſquare Inches; to which add twice 664.32, the Areas of the two Baſes, the Sum is 18608.64, the Area of the Whole, which is 129.22 Feet.

```
     180
      96
    ----
    1080
   1620
   -----
   17280
     664.32
     664.32
   -----------
144)18608.64(129.22
    ----
     420
    1328
      326
       384
       ---
        96
```

The ſuperficial Content of the whole Solid is 129.22 Feet.

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180; that Extent will reach from 96 to 120 Feet. Then, to find the Area of the Baſe, extend the Compaſſes from

144 to 13.84; that Extent will reach from 48 to 4.6 Feet; and 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the ſuperficial Content, as before.

The Demonſtration of thoſe laſt Solids will be the ſame as in the firſt Section; for as in that, ſo in theſe, the Area of the Baſe is multiplied into the Length to find the Content, and the ſame Reaſon is given for one as for the other.

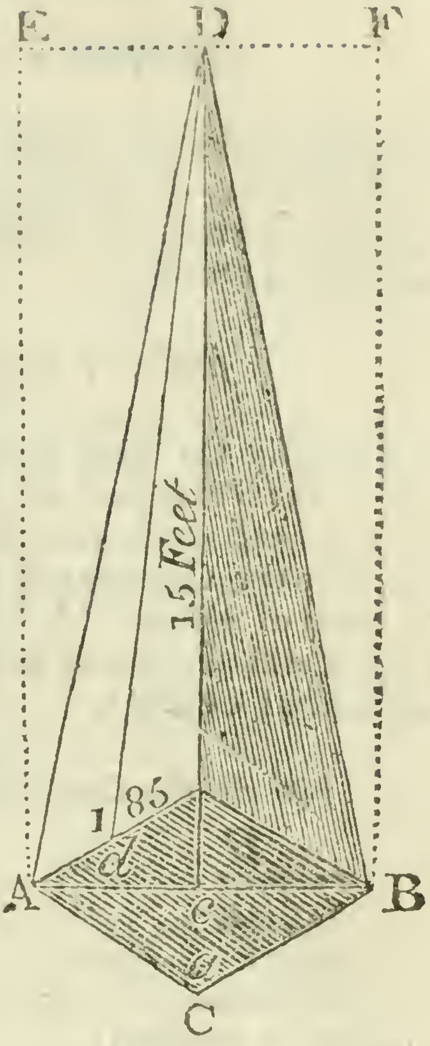
§ IV. *Of a* PYRAMID.

A Pyramid is a ſolid Figure, the Baſe of which is a Polygon, and its Sides plain Triangles, their ſeveral Tops meeting together in one Point. To find the ſolid Content of it, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the Altitude, or Length; and the Product is the ſolid Content of the Pyramid.

Let



Let A B D be a ſquare Pyramid, each Side of the Baſe being 18 5 Inches, and the perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Baſe in Inches; which, multiplied by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the ſolid Content.

```
    18.5
    18.5
    ----
     925
   1480
   185
   ------
  342.25  Area of the Baſe.
       5
   ------
144)1711.25(11.88 Content.
```

```
F.  I.  Pts.
1   6   6
1   6   6
----------
1   6   6
    9   3
        9   3
--------------
2   4   6   3
            5
--------------
11  10  7   3
```

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 18.5 Inches, that Extent, turned twice over from 5 Feet (a third Part of the Height), will fall at laſt upon 11.88 Feet, the ſolid Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe 37, and the Product is 6668.88; which divided by 144, the Quotient is 46.31 Feet, the ſuperficial Content of all but the Baſe; then to that add 2.38 Feet, the Baſe, and it makes 48.69 Feet, the whole Superficial Content.

```
     180.24 the ſlant Height c D.
         37                              144)342.25(2.38
     ------                                  288
     126168                                  ---
     54072                                   542
     -------                                 432
144)6668.88(46.31                            ---
     576      2.38                           1105
     ---      ----                           1152
      908     48.69 the whole Content.
      864
      ---
       448
       432
       ---
        168
        144
        ---
         24
```

By

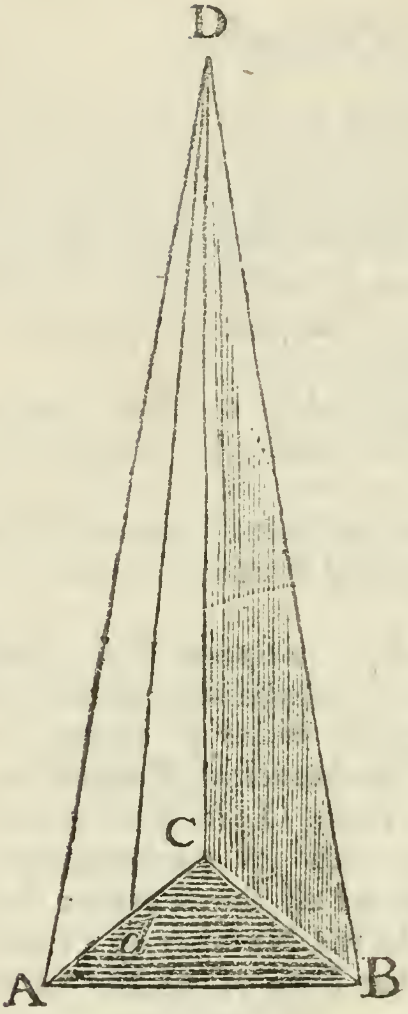
By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compaſſes from 144 to 18.5; (one Side of the Baſe), that Extent will reach from 18.5 to 2.38 *ferè*: Which added to the other, the Sum is 48.69, the whole Superficies.

Demonſtration. Every Pyramid is a third Part of the Priſm, that hath the ſame Baſe and Height (by *Eucl.* 12.7.)

That is, the ſolid Content of the Pyramid ABD (in the laſt Figure) is one third Part of its circumſcribing Priſm ABEF.

For every Pyramid that hath a ſquare Baſe (ſuch as Aa Bb in the laſt Figure) is conſtituted of an infinite Series of Squares, whoſe Sides or Roots are continually increaſing in arithmetical Progreſſion, beginning at the Vertex or Point D, its Baſe Aa Bb being the greateſt Term, and its perpendicular Height CD is the Number of all the Terms: But the laſt Term multiplied into the Number of Terms, the Product will be triple the Sum of all the Series (by Lemma 3.); conſequently $\frac{NLL}{3} = S$. And S is equal to the ſolid Content of the Pyramid. From hence it will be eaſy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumſcribing Priſm (that is, of a Priſm of equal Baſe and Altitude), what Form ſoever its Baſe is of; *viz.* whether it be ſquare, triangular, pentangular, *&c.* You may very eaſily prove a triangular Pyramid to be a third Part of a Priſm of equal Baſe and Altitude, by cutting a triangular Priſm of Cork, and then cut that Priſm into three Pyramids, by cutting diagonally, as I have ſeveral times done, to ſatisfy myſelf and others.



Let ABCD be a triangular Pyramid, each Side of the Baſe being 21.5 Inches, and its perpendicular Height 16 Feet; the Content, ſolid and ſuperficial, is required.

Firſt, find the Area of the Baſe, by multiplying half the Side by the Perpendicular, let fall from the Angle of the Baſe to the oppoſite Side; which Perpendicular will be found to be 18.62; the Half of it, 9.31, multiplied by 21.5, the Product is 200.165 Inches, the Area of the Baſe. Then, becauſe the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52; which divided by 144, the Quotient is 7.41 Feet, the ſolid Content.

			F.	I.	Pts.	
9.31	half the Perpend.					
21.5	the Side,	Side	1	9	6	
		Half Perp.		9	4	
4655						
931			1	4	1	6
1862					7	2
3)200 165	Area Baſe.	Area Baſe	1	4	8	8
						4
66.72	a third Part.					
16	Height.		5	6	10	8
						4
40032						
6672			3)22	3	6	8
144)1067.52(7.41	Solid Cont.	Cont.	7	5	2	2
1008						
595						
576						
192						
144						
48						

In caſting this up by Feet and Inches, inſtead of multiplying by 16, the Height, I break 16 into two ſuch Numbers, as, being multiplied together, the Product may be 16; *viz.* into 4 and 4, and multiply firſt by one, and then the other; a third Part of the laſt Product is the Content.

By Scale and Compaſſes.

Firſt, find a geometrical mean Proportional (as before directed), by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional ſought. Then extend the Compaſſes from

12 to 14.15, that Extent (turned twice over from 16 Feet) will fall at laſt upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

```
                 192.1 Inches, the ſlant Height dD.
Half Periph. 32.25=21.5+10.75
            ------
              9605
             3842
            3842
           5763
           --------
           6195.225 Inches, the Area of all but the
            200.165 Area of the Baſe add.      (Baſe.
           --------
       144)6395.390(44.41 Feet, the whole Content.
           576
           ---
            635
            576
            ---
             593
             576
             ---
              179
              144
              ---
               35
```

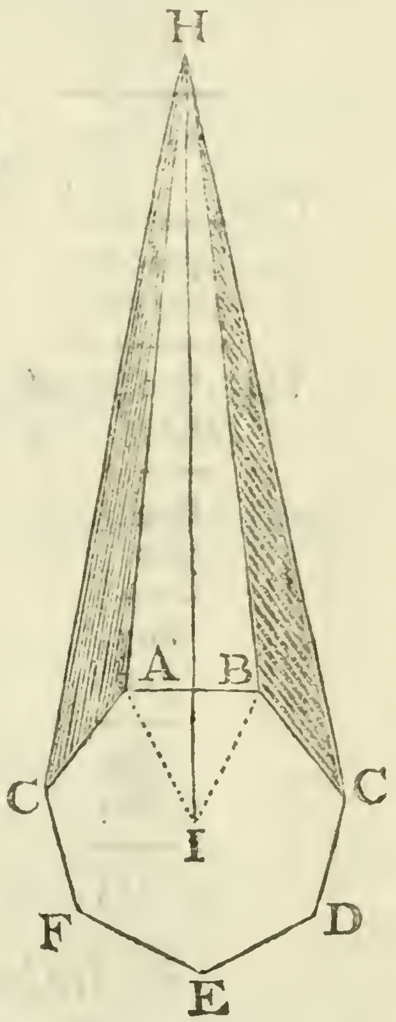
By Scale and Compaſſes.

Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

And

And extend the Compasses from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Base; which, added to the other, makes 44.41 Feet, the Content of the whole.

Let ABCDEFGH be a Pyramid, whose Base is a Heptagon, each Side of it being 15 Inches, the Perpendicular of the Heptagon 15.58 Inches, and the perpendicular Height of the Pyramid, HI, 13.5 Feet; the Content, solid and superficial, is required.



Multiply 15.58 (the Perpendicular) by 52.5 (half the Sum of the Sides of the Heptagon) and the Product is 817.95; which multiplied by 4.5, *viz.* $\frac{1}{3}$ of the Height, and the Product is 3680.775.

Then

Then divide this last Product by 144, and the Quotient is 25.56 Feet, the Content.

```
      15.58 the Heptagon's Perpend.
       52.5 the Half Sum of the Sides.
      -----
       7790
      3116
     7790
     ------
     817.950
         4.5 a third Part of the Height.
     -------
     4089750
    3271800
    -------
144)3680.7750(25.56 Solid Feet.
    288
    ---
     800
     720
     ---
      807
      720
      ---
       877
       864
       ---
        13
```

By Scale and Compasses.

First, find a geometrical mean Proportional between 15.58 and 52.4 (as is before directed), which you will find to be 28.06; then extend the Compasses from 12 to 28.06, that Extent will reach from 4.5 (twice turned over) to 25.56 Feet.

To find the Superficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Baſe 162.75 Inches, by the Half-Sum of the Sides 52.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

```
          162.75                    144)817.95(5.6
            52.5                        ------
          ------                         979
           81375                         1155
          32550                          ----
         81375                              3
        -------
  144)8544.375(59.335  Feet.
      ----       5.68  Baſe add.
      1344     ------
       483     65.015  the whole Content.
        517
         855
        ----
         135
```

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 162.75; that Extent will reach from 52.5 to 59.335 Feet.

And extend the Compaſſes from 144 to 15.58, the Perpendicular of the Heptagon, that Extent will reach from 5.25 to 5.68 Feet, the Content of the Baſe; which add to the former, the Sum is 65.015, the whole ſuperficial Content.

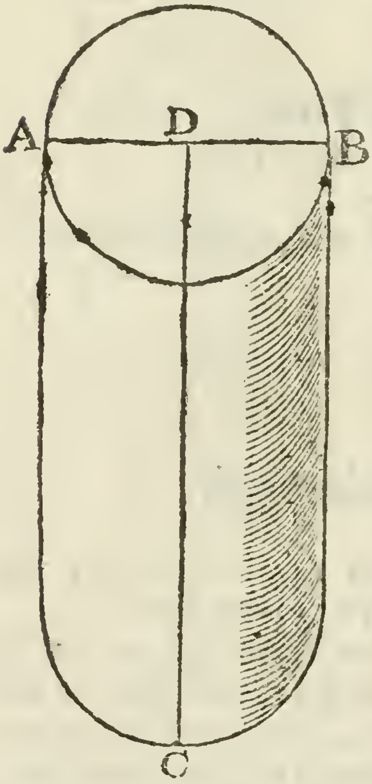
§ V.

§ V. *Of a* CYLINDER.

A Cylinder is a round Solid, having its Bafes circular, equal, and parallel, in Form of a Rolling-ftone ufed in Gardens. To find the folid Content of it, this is

The RULE.

Multiply the Area of the Bafe by the Length, and the Product is the folid Content.



Let A B C be a Cylinder, whofe Diameter A B is 21.5 Inches, and the Length C D is 16 Feet; the folid Content is required.

Firft, fquare the Diameter 21.5, and it makes 462.25; which multiplied by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8184. Divide this laft Product by 144, and the Quotient is 40.34 Feet, the folid Content.

By Scale and Compaffes.

Extend the Compaffes from 13.54 to 21.5, the Diameter, that Extent (turned twice over from 16, the Length) will at laft fall upon 40.34, the folid Content.

To

To find the fuperficial Content.

Firft (by Chap. I. Sect. IX. Prob. 2.), find the Circumference of the Bafe 67.54, which multiplied by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the curve Surface; to which add 5.04 Feet, the Sum of the two Bafes, and the Sum is 95.09 Feet, the whole fuperficial Content.

67.54		363.05
16		2
40524		144)726.10(5.04
6754	90.05 } add.	610
12)1080.64	5.04 }	34
90.05	95.09	

By Scale and Compaffes.

Extend the Compaffes from 12 to 67.54, the Circumference), that Extent will reach from 16 (the Length) to 91.05 Feet, the Curve Surface.

And extend the Compaffes from 12 to 21.5 (the Diameter), that Extent (turned twice from .7854) will at laft fall upon 2 52 Feet, the Area of the Bafe; which doubled is 5.04; this, added to the curve Surface, makes 95.09 Feet, the whole fuperficial Content.

Demonftration. The folid Content of every Cylinder is found, by multiplying the Area of its Bafe into its Height, as aforefaid: For every right Cylinder is only a round Prifm, being conftituted of an infinite Series of equal Circles; that of its Bafe, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Bafe AB, being multi-

multiplied into CD, will be its Solidity (by Lemma I.) Let D = AB, H = CD.

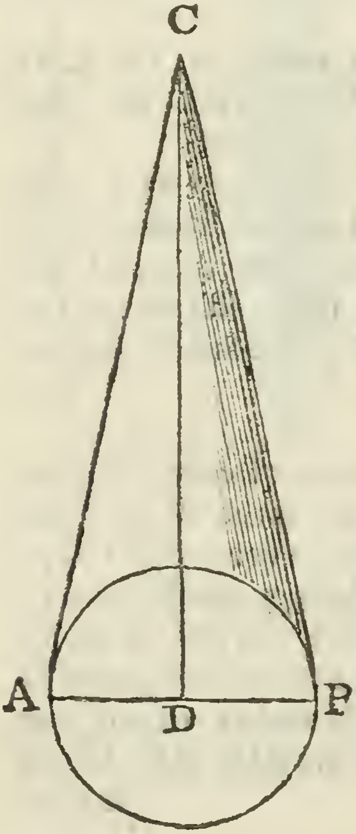
Then $.7854\ DD \times H =$ its Solidity.

§ VI. *Of a* Cone.

A Cone is a Solid, having a circular Baſe, and growing ſmaller and ſmaller, till it ends in a Point which is called the Vertex, and may be nearly repreſented by a Sugar-loaf. To find the Solidity of it, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the perpendicular Height, and the Product is the ſolid Content.



Let A B C be a Cone, the Diameter of whoſe Baſe AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: Firſt, ſquare the Diameter 26.5, and it is 702.25, which multiply by .7854, and the Product is 551.54715; which multiply by 5.5, and the Product is 3033.50932; which divided by 144, the Quotient is 21.07 *ferè*, the ſolid Content of the Cone.

```
        26.5   the Diameter.
        26.5
        ----
        1325
       1590
       530
      ------
      702.25   the Square.
       .7854
      ------
      280900
     351125
    561800
   491575
   ---------
   551.54|715  Area of the Baſe.
      5.5      a third Part of the Height.
   ---------
   275770
  275770
  ---------
144)3033.47|0(21.06 Feet, the Content.
   ---
   153
     947
     ---
      83
```

By Scale and Compaſſes.

Extend the Compaſſes from 13.54 to 26.5, the Diameter, that Extent turned twice over from 5.5 (a third Part of the Height), it will at laſt fall upon 21.06 Feet, the Content.

To find the ſuperficial Content.

Multiply half the Circumference 41.626 by the ſlant Height A C 198.46, and the Product is 8261.09596; which divided by 144, the Quotient is 57.37 *ferè*, the curve Surface; to which add the Baſe, the Sum is 61.2, the ſuperficial Content.

```
           41.626  half Circumference of the Baſe.
           198.46  the ſlant Height.
           ------
           249756
          166504
         333008
        374634
        41626
       -----------
144)8261.09596(57.37  Feet ferè.
    ----         3.83  the Baſe add.
    1061         -----
      530        61.20  the whole Content.
       989
    -----
144)551.54(3.83
    ----
    1195
      434
     ----
        2
```

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 198 46, that Extent will reach from 41.626 to 57.37 Feet, the curve Surface.

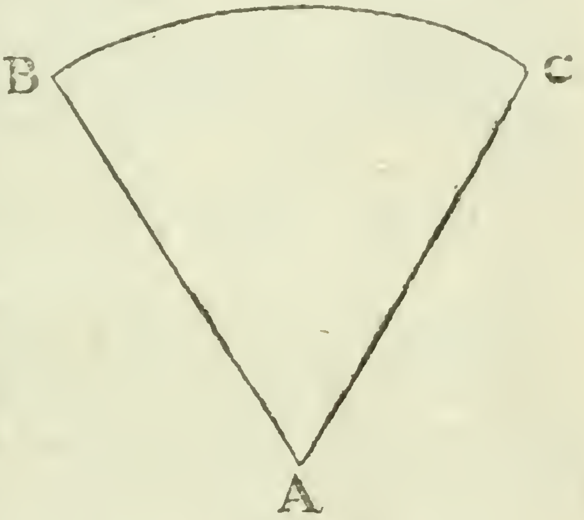
And extend the Compaſſes from 12 to 26.5, the Diameter; that Extent, turned twice over from .7854, will at laſt fall upon 3.83 Feet, the Baſe; which added to 57.37, the Sum is 61.2 Feet, the ſuperficial Content.

Demonſtration. Every Cone is the third Part of a Cylinder of equal Baſe and Altitude. The Truth of this may eaſily be conceived, by only conſidering, that a Cone is but a round Pyramid; and therefore it muſt needs have the ſame Ratio to its circumſcribing Cylinder, as the ſquare Pyramid hath to its circum-

circumfcribing Parallelopipedon; *viz.* as 1 to 3. However, to make it yet clearer, let it be farther confidered, That

Every right Cone is conftituted of an infinite Series of Circles, whofe Diameters do continually increafe in arithmetical Progreffion, beginning at the Vertex, or Point C, the Area of its Bafe AB being the greateft Term, and its perpendicular Height DC, the Number of all the Terms; therefore the Area of the Circle of the Bafe, multiplied by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by Lemma III.

The curve Superficies of every right Cone, is equal to half the Rectangle of the Circumference of its Bafe into the Length of its Side.



For the curve Surface of every right Cone is equal to the Sector of a Circle, whofe Arch BC is equal to the Periphery of the Bafe of the Cone, and Radius AB equal to the flant Side of the Cone: Which will appear very evident, if you cut a Piece of Paper in the Form of a Sector of a Circle, as ABC, and bend the Sides AB and AC together, till they meet, and you will find it to form a right Cone.

I have omitted the Demonftrations touching the Superficies of all the foregoing Solids, becaufe I thought it needlefs, they being all compofed of Squares, Parallelograms, Triangles, &c. which Figures are all demonftrated before. And if the Area

P 2 of

of all ſuch Figures as compoſe the Surface of the Solid, be found ſeverally, and added together, the Sum will be the ſuperficial Content of the Solid.

§ VII. *Of the Fruſtum of a* PYRAMID.

A Fruſtum of a Pyramid is the remaining Part, when the Top is cut off by a Plane parallel to the Baſe. To find the ſolid Content of which there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum, being multiplied into One-third Part of the Fruſtum's Height, will give its Solidity, if the Baſes be ſquare.

Or thus; which is the ſame in Effect:

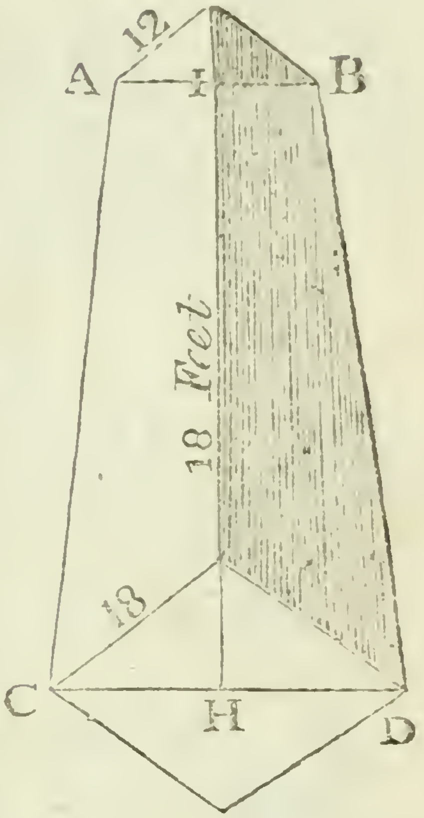
Multiply the Areas of the two Baſes together, and to the ſquare Root of the Product add the two Areas; that Sum, multiplied by One-third of the Height, gives the Solidity of any Fruſtum, ſquare or mult-angled.

RULE II.

To the Rectangles of the Sides of the two Baſes, add One-third Part of the Square of their Difference; that Sum, being multiplied into the Height, will produce the Solidity, if the Baſes be Squares: But if they be triangular or multangular, the ſaid Rectangle of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the ſquare Root of it will be ſuch a mean

mean Side as will reduce the tapering Solid to a Priſm equal to it.

Example. Let A B C D be the Fruſtum of a ſquare Pyramid, the Side of the greater Baſe 18 Inches, and the Side of the leſſer 12 Inches, and the Height 18 Feet; the Solidity is required.



Firſt, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, the Square of which is 36, a third Part of this is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Baſe; this being multiplied by 18 Feet, the Length, the Product is 4104; and this being divided by 144, the Quotient is 28.5 Feet, the Content.

Or, by the firſt Rule, thus: the Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of theſe three is 684, which being multiplied by 6, the Product is 4104; and divided by 144, the Quotient is 28.5 Feet, the ſame as before.

See the Work both Ways.

18	6 Diff.	18	12
12	6	18	12
216	3)36 Square.	324 Sq.	144 Sq.
12	add—	144	
	12 a Third.	216	
228 the Sum.		684 the Sum.	
18 the Height.		6 a 3d of the Height.	
1824		144)4104(28.5 Feet.	
228		1224	
144)4104(28.5		720.	
1224		...	
720			
...			

By Feet and Inches, thus:

	F.	I.	I.
Mult.	1	6	6
by	1		6
Prod.	1	6	3)36q.
add	0	1	12
Mult.	1	7	
	18	0	Height.
	18	0	
	9	0	
	1	6	
Cont.	28	6	

Or thus;

F.	I.	
2	3	Sq. of the greater.
1	6	the Rectangle.
1	0	Square of the leſs.
4	9	Trip. of a mean Ar.
6	0	a 3d of the Height.
28	6	

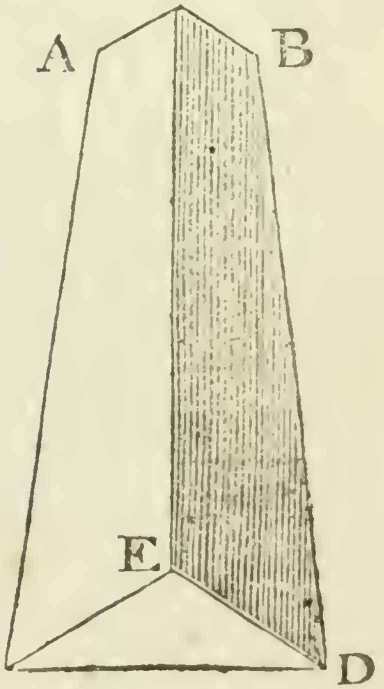
To

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 48; add both the Perimeters together, the Sum is 120; the half of it is 60; which multiplied by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Baſes 2.25 Feet, and 1 Foot; the Sum is 93.25 Feet, the whole ſuperficial Content.

18	12	18 the Height.
4	4	60
72	48	12)1080
48		90 Feet.
2)120		2.25 the greater Baſe.
60		1 the leſſer Baſe.
		93.25 the Sum.

Again: Let ABC be the Fruſtum of a triangular Pyramid, each Side of the greater Baſe 25 Inches, and each Side of the leſſer Baſe 9 Inches, and the Length 15 Feet; the ſolid Content of it is required.



By the ſecond Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16, which ſquared, makes 256; a third Part of this is 85.333, which added to 225, the Sum is 310.333; and this multiplied by 433, the Product is 134.374, &c. which is the Area of a mean

Baſe;

Baſe; and that multiplied by 15 Feet, the Length, the Product is 2015.61; which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the firſt Rule: Find the Area of the greater Baſe, which you will find to be 270.625, and the Area of the leſſer Baſe will be 35.073; theſe two Areas multiplied together, the Product is 9491.630625; the Square Root of which is 97.425; to which add the two Areas, and the Sum is 403.123; which multiplied by a third Part of the Length, 5, the Product is 2015.615; and that divided by 144, the Quotient is 13.99 Feet, as before.

See the Working of both.

```
         25      25
          9       9
Product 225      16  Diff.
                 16
                 96
                16
             3)256   the Square.
                85.333   a third Part.
               225       add.
               310.333
                   433   tabular Number, p. 89.
               930999
              930999
            1241333
            134.374189  mean Area.
                    15  Length.
             671870945
            134374189
    144)2015.61|2835(13.99 Feet.
           575
           1436
            1401
             105
```

```
  25
  25
 ----
 125
 50
 ----
 625 Square.
·433
 ----
 1875
1875
2500
 ----
270.625 Area.
```

```
 9
 9
--
81 Square.
```

```
     ·433
       81
    -----
      433
    3464
   ------
   35 073

  270.625
   35·073
  -------
   811875
  1894375
 13531250
 811875
---------
9491.630625
 .  .  .  .  .

9491.630625(97·425
81
---
187)1391
    1309
    ----
1944)8263
     7776
     ----
19482)48706
      38964
      -----
194845)974225
       974225
       ------
       ......
```

270.625	greater Area.
97.425	the mean Proportional.
35.073	the leffer Area.
403.123	the Triple of a mean Area.
5	a third Part of the Height.

144)2015 615(13.99 Feet, the Solidity.
575
1436
1401
105

In finding the Area of the triangular Bafe, I multiply by 433, becaufe that is the Area of the equilateral Triangle, when the Side of it is 1. A Table of the Areas, or Multipliers, for finding the Areas of Polygons, you'll find in *p.* 89.

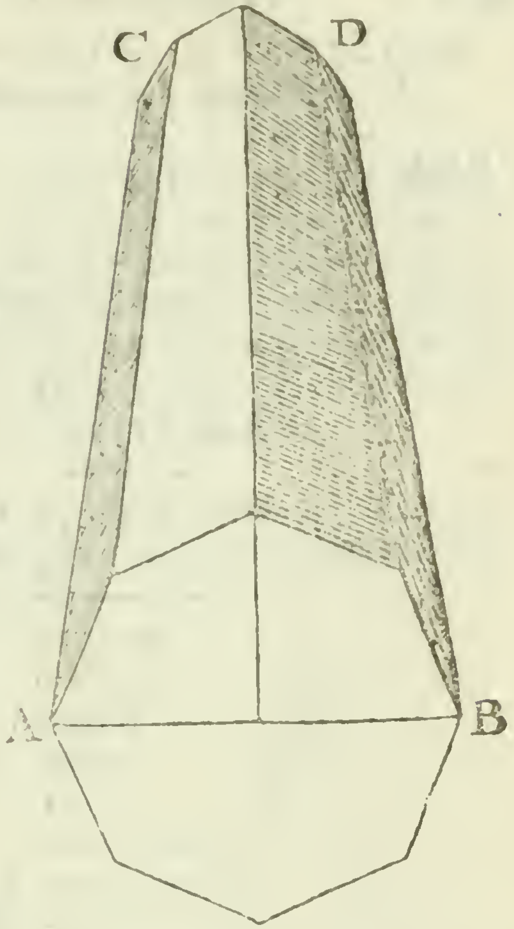
Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the fuperficial Content.

The Perimeter of the greater Bafe is 75, and the Perimeter of the leffer Bafe is 27; the Sum of both is 102, and the half Sum is 51; which multiplied by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63 75; to which add the Sum of the two Bafes 2.12 Feet, and the Sum is 65.87 Feet, the whole fuperficial Content.

Note, That 51 fhould have been multiplied by the flant Height, but the Difference it would make is but .06 of a Foot, which is inconfiderable.

Again:



Again: Suppofe ABCD to be the Fruftum of a Pyramid, having an octagonal Bafe, each Side of it being 9 Inches, and each Side of the leffer Bafe 5 Inches, and the Length, or Height, 10.5 Feet; the Solidity is required.

By the fecond Rule, multiply the greater Side 9 by the leffer Side 5, and the Product is 45; then the Difference between 9 and 5 is 4; which fquared makes 16; a third Part of which is 5.3333, which added to 45, the Sum is 50.3333; multiply this laft by the Number in the Table 4.8284, and the Product is 243.0292, the Area of a mean Bafe; which multiplied by the Height 10.5 Feet, the Product is 2551.8066; then divide this laft Product by 144, and the Quotient is 17.72 Feet, the folid Content.

See the Work.

Mult.

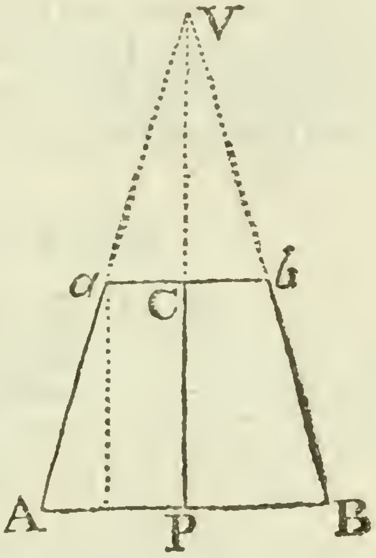
```
Mult. 9 Inches.   9 from the greater Side.
   by 5 Inches.   5 ſubtract the leſſer.
      ---         -
Prod. 45          4
                  4
                 --
              3)16 ſquare.
                 --
                 5.3333 a third Part.
           Add  45
                -------
           Sum  50.3333 the Sq. of a mean Side.
                 4.8284 tabular Number, p. 89.
                -------
                2013332
                 402666
                  10067
                   4026
                    201
                -------
               243.0292 a mean Area.
                   10.5 the Height.
               --------
               12151460
              24302920
              ----------
       144)2551.80|660(17.72
           144
           ----
           1111
           1008
           ----
            1038
            1008
            ----
              300
              288
              ---
               12
```

To

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 40, and their Sum is 112; the Half of it is 56, which multiplied by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Baſes, 3.55, and the Sum is 52.55 Feet, the whole ſuperficial Content.

Demonſtration. From the Rules delivered in the IVth and VIth Sections, the two foregoing Rules may eaſily be demonſtrated.



Suppoſe a Square Pyramid, ABV, to be cut by a Plane at *a* b, parallel to its Baſe AB, and it were required to find the Solidity of the Fruſtum, or Part *a* b AB. Let there be given

D = BA, the Side of the greater Baſe.
d = b *a*, the Side of the leſſer Baſe.
H = CP, the Perpendicular Height.

First, | 1 | $D - d : H :: d : \frac{dH}{D-d} = VC$ by the Figure.
Then | 2 | $DD \times \frac{H + VC}{3} =$ the whole Pyramid BVA, by Section the IVth.
And | 3 | $dd \times \frac{1}{3} VC =$ the Pyramid *a* V b cut off.

Q

Then,

Then, in the 2d and 3d Steps, if, inftead of VC, you take $\frac{dH}{D-d}$, equal to it, by the firft Step, it will be,

viz. 1. 2.	4	$\frac{DDDH}{3D-3d}$ = the whole Pyramid BVA;
and 1. 3.	5	$\frac{dddH}{3D-3d}$ = the Pyramid *a* V b.
4—5.	6	$\frac{DDDH-dddH}{3D-3d}$ = the Fruftum *a*b A B.

And by dividing DDD—ddd by D—d, and then multiplying the Quotient by $\frac{1}{3}$ H, the laft Step will be reduced to $DD+Dd+dd \times \frac{1}{3}H$ = the Fruftum *a* b A B; which, in Words, is thus:

To the Rectangle of the Sides of the two Bafes add the Sum of their Squares; that Sum being multiplied into One-third of the Fruftum's Height, will give its Solidity; which is the fame as the firft Rule of this Section.

See the Work of the Divifion.

$$
\begin{array}{rl}
D-d) & DDD-ddd\ (DD+Dd+dd \\
& DDD-DDd \\
\hline
& \quad DDd-ddd \\
& \quad DDd-Ddd \\
\hline
& \quad\quad Ddd-ddd \\
& \quad\quad Ddd-ddd \\
\hline
& \quad\quad 0 \quad 0
\end{array}
$$

The

The ſame Reaſon will hold good for all Fruſtums of Pyramids or Cones, whether the Baſe be triangular or multangular, becauſe the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles, are proportional to the Area; which prove the latter part of the ſaid firſt Rule.

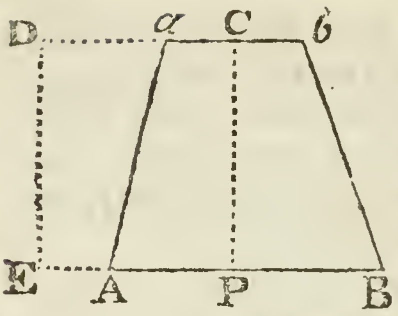
Again, to prove the ſecond Rule.

Suppoſe	1	$x = D - d$. And $F =$ the Fruſtum.
then	2	$DD + Dd + dd = \frac{3F}{H}$ by the laſt.
1 2	3	$xx = DD - 2Dd + dd$.
2 — 3	4	$3Dd = \frac{3F}{H} - xx$.
4 — 3	5	$Dd - \frac{F}{H} = -\frac{1}{3}xx$. Or $Dd + \frac{1}{3}xx = \frac{F}{H}$
5 × H	6	$\overline{Dd + \frac{1}{3}xx} \times H = F$, the Fruſtum *a* b A b.

Which, in Words, is thus:

To the Rectangle of the Sides of the two Baſes add one third Part of the Square of the Difference of the ſaid Sides, and multiply the Sum by the Height of the Fruſtum, the Product is the Solidity.

The ſuperficial Contents of Fruſtums (all but the Baſes) are compoſed of as many Trapeziums, as the Fruſtum has Sides. Thus, the ſquare Fruſtum *ab* AB, in the laſt Figure, is compoſed of four Trapeziums, having the two upper, and alſo the two lower Angles equal; if, therefore, the Trapezium *a*b AB be cut in two by the Line CP, and the two Pieces laid together, the Line bB upon the Line *a*A, the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure annexed; the Parallelogram



DCEP being equal to the Trapezium *ab*AB; becauſe the Side D*a* is equal to PB, and EA is equal to *a*C. Therefore, to find the Area of the Trapezium, add half the Side *ab* to half the Side AB, and it makes DC or EP; which multiplied by the Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium *ab*AB; then, if that be multiplied by the Number of Trapeziums, the Product will be the ſuperficial Content of the Fruſtum, wanting the Baſes. Or, if the whole Perimeter of the greater Baſe be added to the Perimeter of the leſſer Baſe, and half the Sum multiplied by the Height, the Product will be the ſuperficial Content of all the Trapeziums at once.

Note, That half the Sum of the Perimeters ſhould be multiplied by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing Examples I have multiplied by the perpendicular Height, becauſe the Difference is generally very inconſiderable: But the ſlant Height may always be taken with leſs Trouble than the perpendicular Height, and is therefore always given in Practice.

§ VIII. *Of the Fruſtum of a* Cone.

A Fruſtum of a Cone, is that Part which remains when the top End is cut off by a Plane parallel to the Baſe. To find the ſolid Content, the Rules are the ſame in Effect as for the Fruſtum of a Pyramid.

RULE I.

To the Rectangle of the Diameters of the two Baſes add the Squares of the ſaid Diameters, and multiply

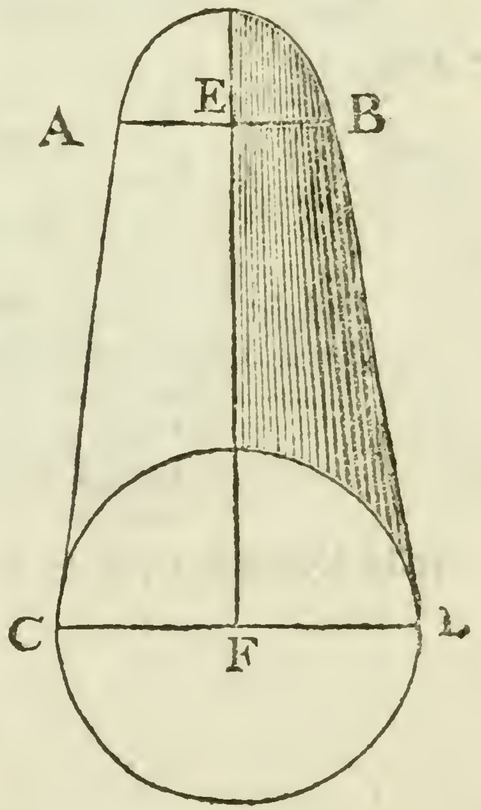
multiply the Sum by .7854, the Product will be the Triple of a mean Area; which multiplied by $\frac{1}{3}$ of the perpendicular Height, that Product will be the ſolid Content.

Or thus: Multiply the Areas of the greater and leſſer Baſes together, and out of the Product extract the Square Root, and add the two Areas and Square Root together, and multiply the Sum by one-third of the perpendicular Height, the Product is the ſolid Content.

RULE II.

To the Rectangle of the greater and leſſer Diameters, add one-third Part of the Square of the Difference, and multiply the Sum by .7854, the Product is a mean Area; which multiplied by the Perpendicular Height, the Product is the Solidity.

Example. Let ABCD be the Fruſtum of a Cone, whoſe greater Diameter CD is 18 Inches, and the leſſer Diameter AB 9 Inches, and the Length 14.25 Feet, the ſolid Content is required.



Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, the Square of which is 81; a third Part is 27, which add to 162, the Sum is 189; this multiplied by .7854, the Product is 148.44; which divided by 144, the Quotient is 103 Feet, the Area of a mean Baſe; which

multiplied by 14.25 Feet, the Height, the Product is 14.6775 Feet, the ſolid Content.

Or thus, by the firſt Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or the Product of 18 by 9, is 162; the Sum of theſe three is 567, which multiplied by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this multiplied by 4.75 Feet (a third Part of the Height), and the Product is 14.6775 Feet, the Solidity, the ſame as before.

See the Work.

	18	18	from	.7854
	9	9	ſubtr.	189
	162	9	Rem.	70686
Add	27	9		62832
				7854
Sum	189	3)81	Square.	
				144)148.4406(1.03
		27	a Third.	144
Height		14.25	Feet.	444
Area Baſe		1.03	Feet.	432
		4275		12
		14250		

Solid Content 14.6775 Feet.

324 the Square of 18.
162 the Rectangle.
81 the Square of 9.

567 the triple Square of a mean Diameter.

```
          ·7854
            567
          -----
          54978
         47124
        39270
       --------
  144)445·32|18     (3·09
      -------         4·75
        1332        -----
        ----          1545
          36         2163
                    1236
                    -----
The Solidity        14 6775
```

To find the Superficial Content.

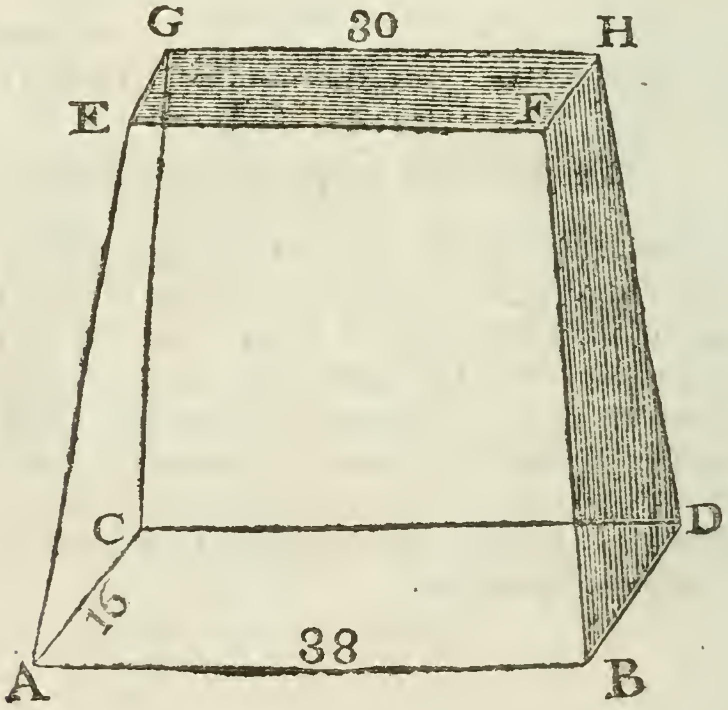
By Chap. I. Sect. IX. Problem 2. you will find the Circumference of the greater Baſe to be 56.5488, and of the leſſer Baſe 28.2744; the Sum of both is 84.8232; the Half Sum is 42.4116; which multiplied by 14.25 Feet, and the Product is 604.36, &c. which divided by 12, the Quotient is 50.36 Feet, the curve Surface; to which add the Sum of the two Baſes, 2.21 Feet, the Sum is 52.75 Feet, the whole ſuperficial Content.

§ IX. *To meaſure the Fruſtum of a rectangled* Pyramid, *called a* PRISMOID, *whoſe Baſes are parallel one to another, but diſproportional.*

The RULE.

TO the greateſt Length add half the leſſer Length, and multiply the Sum by the Breadth of the greater Baſe, and reſerve the Product.

Then, to the leſſer Length, add half the greater Length, and multiply the Sum by the Breadth of the leſſer Baſe, and add this Product to the other Product reſerved, and multiply that Sum by a third Part of the Height, and the Product is the ſolid Content.



Example.

Example. Let ABCDEFGH be a Priſmoid given, the Length of the greater Baſe AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the leſſer Baſe EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the ſolid Content is required.

To the greater Length AB 38, add half EF the leſſer Length 15, the Sum is 53; which multiplied by 16, the greater Breadth, and the Product is 848; which reſerve.

Again, to EF 30, add half AB 19, and the Sum is 49; which multiplied by 12 (the leſſer Breadth EG), the Product is 588; to which add 848 (the reſerved Product), and the Sum is 1436; which multiplied by 2 (a third Part of the Height), and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the ſolid Content.

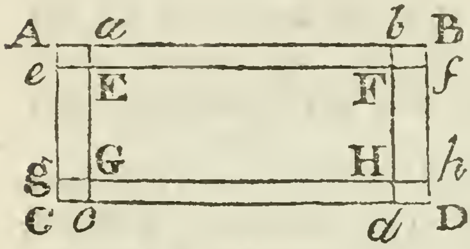
38 = AB	30 = EF
15 = $\frac{1}{2}$EF	19 = $\frac{1}{2}$AB
53	49
16 = AC	12 = EG
318	588
53	
848	
588	
1436	
2 = a third Part of the Height.	

144)2872(19.94 Feet, the Content.

1432
1360
640

64

To prove this Rule. Let us ſuppoſe the Solid cut into Pieces, ſo as to make it capable of being meaſured by the foregoing Rules; thus: Let ABCD repreſent the greater Baſe, and EFGH the leſſer Baſe; and let the Solid be ſuppoſed to be cut through by the Lines, *ac*, *bd*, and *ef*, *gh*, from the Top to the Bottom; ſo will there be a Parallelopipedon, having its Baſes equal to the leſſer Baſe EFGH, and its Height 6 Feet, equal to the Height of the Solid: Multiply 30 (the Length of the Baſe



by 12, the Breadth thereof), and the Product is 360; which multiplied by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whoſe Baſes are *ab*EF, and GH*cd*; if theſe two Pieces be laid together, the thick End of one to the thin End of the other, they will compoſe a rectangled Parallelopipedon; which to meaſure, multiply the Length of the Baſe 30 by its Breadth 2, and the Product is 60; which multiplied by 6 (the Height), the Product is 360. Then there are two other Wedge-like Pieces, whoſe Baſes are *e*E*g*G, and *f*FH*h*; theſe two laid together will compoſe a rectangled Parallelopipedon: To meaſure this, multiply the Length of the Baſe 12 by the Breadth 4, the Product is 48; which multiplied by 6 (the Height), the Product is 288. And laſtly, there are four rectangled Pyramids, at each Corner one; which to meaſure, multiply the Length of one of the Baſes 4 by its Breadth 2, the Product is 8; which multiplied by 2 (a third Part of the Height) the Product is 16; and that multiplied by 4 (becauſe there are four of them), the Product is 64. Then add all theſe together, and the Sum is 2872, and divide by 144, the Quotient is 19.94 Feet, the ſame as before; which ſhews the Rule to be true.

See

See the Work.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			64
64			

144)2872(19.94 Feet, the whole Content.

1432
1360
640
64

To find the fuperficial Content.

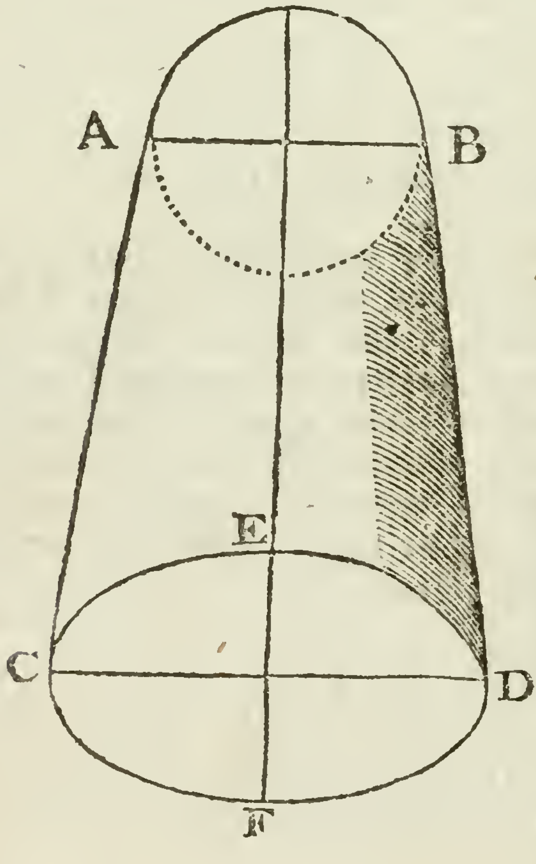
The Sum of the Ends of the two Bafes is 28, which being multiplied by 72.11, the flant Height of each End, is 2019.08 Inches: alfo the Sum of the Sides of the two Bafes is 68; and this multiplied by 72.03, the flant Height of the Sides, gives 4898.04 Inches. To the Sum of thefe two add the Areas of the two Bafes, 608, and 360; their Sum is 7885.12 Inches, which, being divided by 144, gives 54.75 for the whole fuperficial Content.

To meaſure a CYLINDROID; *that is, a Fruſtum of a* Cone, *having its Baſes parallel to each other, but unlike.*

The RULE.

TO the longeſt Diameter of the greater Baſe, add half the longeſt Diameter of the leſſer Baſe, add multiply the Sum by the ſhorteſt Diameter of the greater Baſe, and reſerve the Product.

Then, to the longeſt Diameter of the leſſer Baſe, and half the longeſt Diameter of the greater Baſe, and multiply the Sum by the ſhorteſt Diameter of the leſſer Baſe, and add the Product of the former reſerved Sum, and that Sum will be the triple Square of a mean Diameter; which multiplied by .7854, and that Product multiplied by a third Part of the Height, the Product is the ſolid Content.



Exam. Let ABCD be a Cylindroid, whoſe bottom Baſe is an Oval, the tranſverſe Diameter being 44 Inches; and the conjugate Diameter 14 Inches; and the upper Baſe is a Circle, of which the Diameter is 26 Inches; and the Height of the Fruſtum is 9 Feet; the Solidity is required.

To 44 (the greater Diameter of the lower Baſe) add 13 (half the

the Diameter of the leſſer Baſe, the Sum is 57; which multiplied by 14 (the conjugate Diameter of the greater Baſe) the Product is 798; which reſerve. Then to 26 (the Diameter of the leſſer Baſe) add 22 (half the tranſverſe Diameter of the greater Baſe), and the Sum is 48; which multiplied by 26 (the Diameter of the leſſer Baſe), the Product is 1248; to which add the former reſerved Product, the Sum is 2046; which multiplied by .7854, the Product is 1606.9284; which multiplied by 3 (a third Part of the Height), the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the ſolid Content. See the Work.

44=CD
13=half AB
——
57 Sum.
14=EF
——
228
57
——
798 Product reſerved.

26=AB
22=half CD
——
48 Sum.
26=AB
——
288
96
——
1248
798 add.
——
2046
.7854
——
8184
10230
16368
14322
——
1606.9284
3
——
144)4820.78|52(33.47
——
500
687
1118
——
110

R This

This Rule being the ſame as that in the laſt Section, the Proof of that may ſerve as a ſufficient Proof of this, if what has been before written be well conſidered.

To find the ſuperficial Content.

To the Periphery of the Ellipſis 91.106,* add the Periphery of the Circle 81.682, and the Sum is 172.788; the Half, 86.394, multiplied by 9, the Product is 777.546; which divided by 12, the Quotient is 64.8 Feet, the curve Surface: Then the Area of the Ellipſis is 3.36 Feet, and the Area of the Circle is 3.69 Feet; both which added to the curve Surface, the Sum is 71.85 Feet, the whole ſuperficial Content.

§ XI. *Of a* Sphere *or* Globe.

A Sphere, or Globe, is a round ſolid Body, every Part of its Surface being equally diſtant from a Point within, called its Center; and it may be conceived to be formed by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

The RULE.

1. Multiply the Axis, or Diameter, into the Circumference, the Product is the ſuperficial Content; which multiplied by a ſixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 is to 11, ſo is the Cube of the Axis to the ſolid Content.

3. Or, as 1 is to .5236, ſo is the Cube of the Axis to the ſolid Content.

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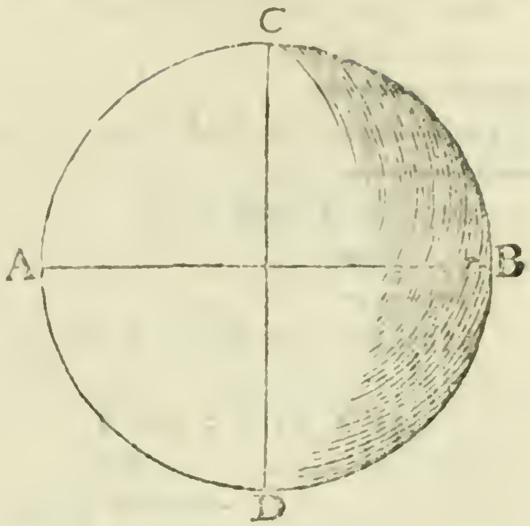
* The Periphery of an Ellipſis is found nearly by multiplying half the Sum of the two Diameters by 3.1416.

Example.



Example. Let ABCD be a Globe, the Axis of which is 20 Inches, then the Circumf. will be 62.832: And, by the firſt Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the ſuperficial Content in Inches; take a ſixth Part of this, which is 209.44, (becauſe an exact ſixth Part of 20 cannot be taken), multiply that ſixth Part by 20 (the Axis), and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the ſuperficial Content by the Axis, and take a ſixth Part of the Product, the Anſwer will be the ſame.

Or thus, by the ſecond Rule:

The Cube of the Axis is 8000: this multiplied by 11, the Product is 88000; which being divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiplied by .5236, the Product is 4188.8, the Solidity, the ſame as by the firſt Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

```
 62.832
     20
-------
6)1256.640 the ſuperficial Content.
----------
  209.44 a ſixth Part.
      20
  -------
  4188.80 the Solidity in Inches.

      21 : 11 : : 8000
                    11
                  ----
          21)88000(4190 47 the Content.
               40
               190
                 100
                  160
                  ---
                   13

    1 : .5236 : : 8000
               8000
       -------------
   1728)4188.8000(2.424 Feet, the Solidity.
        ----
        7328
         4160
          7040
          ----
           128
```

Note, If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016837.

By

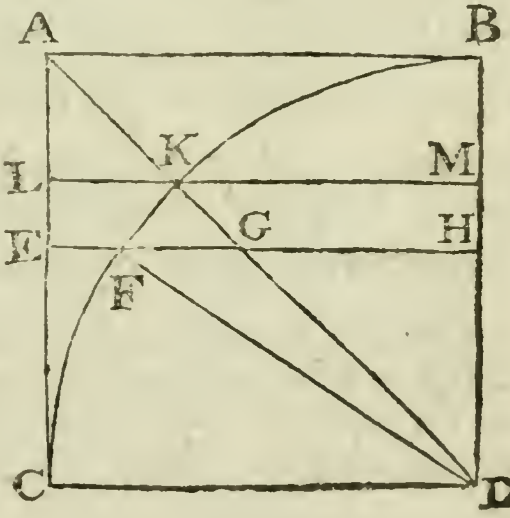
By Scale and Compaſſes.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turned three times over from .5236), will at the laſt fall upon 4188.8, the ſolid Content in Inches: Or, extend the Compaſſes from 1728 to 8000 (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the ſolid Content in Feet.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turned twice over from 3.1416), will at laſt fall upon 1256 64, the ſuperficial Content in Inches: Or, extend the Compaſſes from 144 to 400 (the Square of the Axis), that Extent will reach from 3 1416 to 8 72, the ſuperficial Content in Feet.

Demonſtration. Every Sphere is equal to a Cone, whoſe perpendicular Axis is the Radius of the Sphere, and its Baſe a Plane equal to all the Surface of it.

For you may conceive the Sphere to conſiſt of an infinite Number of Cones, whoſe Baſes, taken altogether, compoſe the Surface, and whoſe Vertexes meet altogether in the Center of the Sphere: Hence the Solidity of the Sphere will be gained, by multiplying its Surface by $\frac{1}{3}$ of its Radius.



Let the Square A B C D, the Quadrant CBD, and the right-angled Triangle ABD, be ſuppoſed all three to revolve round the Line BD as an Axis: Then will the Square generate a Cylinder, the Quadrant a Hemiſphere, and the Triangle a Cone, all of the ſame Baſe and Altitude.

 Then

Then the Square of EH ($= \square$ FD) $= \square$ FH $+ \square$ DH (but DH$=$GH). And ſince Circles are as the Squares of their Diameters (by *Euclid* 12. 2.) the Circle made by the Revolution of FH muſt be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of EH from both, there will remain the Circle made by the Motion of GH, equal to the Ring deſcribed by the Motion of EF. And thus it will always be, where-ever you draw the Line EH or IM, *&c.*

Therefore the Aggregate, or Sum, of all the Rings, made by the Revolution of the EF's, muſt be equal to that of all the Circles made by the Motion of the GH's; *i.e.* the Diſh-like ſolid, formed by the revolving Rings, will be equal to the Cone, formed by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Diſh-like Solid will be as the Cone, $\frac{1}{3}$ of the circumſcribing Cylinder, and conſequently the Hemiſphere muſt be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder.

Let the Radius of the Sphere be $r=$CD, then the Diameter will be $2r$, let the Surface of the Sphere, generated by the revolving Semicircle, be called S, and that of the Cylinder, formed by the Revolution of 2 AC$=2r=$ the Diameter, be called $ſ$. Wherefore in what was juſt now proved, the Expreſſion for the Solidity of the Sphere in this Notation will be $\frac{rS}{3}$; and putting c equal to the Circumference of the Baſe, or for the Periphery of a great Circle of the Sphere, the curve Surface of the Cylinder will be $2rc$, alſo $\frac{rc}{2}$ will be the Area of a great Circle (by Sect. IX. of Chap. I. Prob. 1.) and this multiplied by $2r$, makes rrc; which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, ſince $ſ$ was put equal to $2rc=$ the

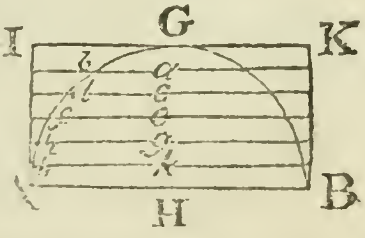
8

the Curve Surface of the Cylinder: $\frac{rſ}{2}$ (by ſubſtituting $ſ$ for $2rc$) will be alſo = the Solidity of the Cylinder. Now, ſince the Sphere is = $\frac{2}{3}$ of the Cylinder, $\frac{rS}{3} = \frac{2 \times ſr}{3 \times 2}$; that is, $\frac{rS}{3} = \frac{2ſr}{6} = \frac{ſr}{3}$ Wherefore $rS = rſ$; that is, dividing by r, $S = ſ$; or the Surface of the Sphere is equal to the curve Surface of the Cylinder, but the curve Surface of the Cylinder was $2rc$.

Wherefore, to find the Area of the Surface of either Sphere or Cylinder, you muſt multiply the Diameter ($= 2r$) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Baſe. From this Notation alſo $\frac{rc}{2}$, the Area of a great Circle of the Sphere, is plainly $\frac{1}{4}$ of $2rc$, the Surface of the Sphere; that is, the Surface of the Sphere is Quadruple of the Area of the greateſt Circle of it.

Wherefore, to $2rc$, the convex Surface of the Cylinder, add rc, the Area of both its Baſes, you will have $3rc$; which ſhews you, that the Surface of the Cylinder (including its Baſes) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the ſame Baſe and Altitude, by Lemma VI. aforegoing, thus:



Let AGB repreſent the Hemiſphere, and AIKB half the Cylinder; then, if the Semidiameter GH be divided into ſix equal Parts, and Lines be drawn parallel to AB, the Diameter, the Squares of the Semichords, ab, cd, ef, &c. will be a Series of Numbers, whoſe greateſt Term AH is a ſquare Number, the other differing

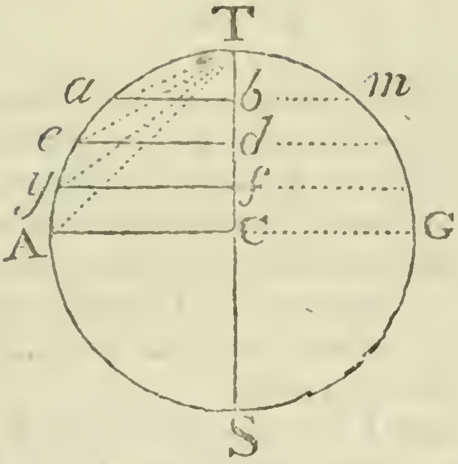
differing by odd Numbers; that is, A H is 36, k l 35, g h 32, e f 27, c d 20, a b 11: But an infinite Series of ſuch Numbers are in Proportion to the infinite Number of Terms, all equal to the greateſt, as 2 to 3. And becauſe the Hemiſphere is compoſed of an infinite Number of Circles, whoſe Diameters are the Chords of the Semicircle; and the Half-Cylinder is compoſed of an infinite Number of Circles, whoſe Diameters are all equal to the Diameter of the Semicircles AB; therefore the Hemiſphere is in Proportion to the Half-Cylinder as 2 to 3; and conſequently the whole Sphere bears the ſame Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greateſt Circle, is thus proved:

The Solidity of the Sphere is conſtituted of an infinite Number of parallel Circles (as is aforeſaid); conſequently the Superficies of the Sphere will be compoſed of the Peripheries of thoſe Circles which conſtitute its Solidity.

Note, In the following Demonſtration, ⊙ ſignifies any Circle in general; and if any two Letters be joined to it, thus, ⊙ A B, *&c.* then it denotes the Area of ſuch a Circle as thoſe two Letters repreſent the Radius of.

Let



Let $D = TS$, the Axis of any Sphere; then, according to the Property of a Circle, it

will be	1	$\overline{D - Tb} \times Tb = \square ab$;
that is,	2	$D \times Tb - \square Tb = \square ab$;
therefore	3	$D \times Tb = \square aT$.
For	4	$\square Ab + \square Tb = \square aT$ (*Eucl.* 1. 47.)
and {	5	$D \times dT = \square eT$.
	6	$D \times Tf = \square yT$.

Hence it is evident, that the Series $\square aT$, $\square eT$, $\square yT$, *&c.* are in the fame Ratio with Tb, Td, Tf, *&c. viz.* in arithmetical Progreffion: Whence it follows, that the $\odot aT =$ to the Sum of all the Circles Peripheries between T and b.

And $\odot eT =$ the Sum of all the Circles Peripheries between T and d, *&c.*

Confequently, that the $\odot AT =$ the Sum of all the Circles Peripheries, included between T and C; that is, $\odot AT =$ the Superficies of the Hemifphere.

And becaufe $\square AC + \square TC = \square AT$, and $\square AC$ is equal to $\square TC$; therefore $\odot AT = 2 \odot AC$, is the Superficies of the Hemifphere.

Confequently, $4 \odot AC$ will be the Superficies of the whole Sphere. Which was to be proved.

Scholium.

Scholium.

From the Method here uſed in proving the whole Superficies, it will be eaſy to find the curve Superficies of any Fruſtum, or Part of a Sphere, that is cut off by a Right Line or Plane; *viz.* ſuch as the Fruſtum a Tm in the laſt Scheme, the curve Superficies of which is ⊙aT, as above. Therefore (becauſe □ab + □Tb = □aT) it will be ⊙ab + ⊙Tb = the curve Superficies of that Fruſtum.

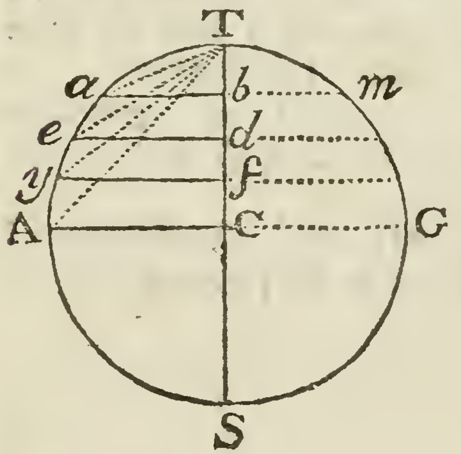
But if the Axis TS, and the Height Tb of the Fruſtum, are given, then it will be TS × Tb = □aT, as in the Third Step above; which gives the Proportion or Theorem following; *viz.*

As the Axis of the Sphere is to the whole Superficies of that Sphere, ſo is the Height of any Fruſtum to its curve Superficies.

To which if there be added the Area of the Fruſtum's Baſe, the Sum will be the whole Superficies of the Fruſtum.

That the Solidity of every Sphere is Two-thirds of its circumſcribing Cylinder, *may be thus proved.*

According to the Work above, it appears that ⊙ab, ⊙ed, ⊙yf, &c. do conſtitute the Solidity of the Sphere; and that □aT, □eT, □yT, &c. are a Series of Terms in arithmetical Progreſſion, □AT being the greateſt Term, and TC the Number of Terms; therefore ⊙AT × $\frac{1}{2}$TC = the Sum of all the Series, by Lemma 2.



And becauſe □aT — □Tb = □ab. □eT — □Td = □ed. □yT — □Tf = □yf. □AT — □TC = □AC, &c. in

in which $\square$ Tb, $\square$ Td, $\square$ Tf, &c. are a Series of Squares, whoſe Roots Tb, Td, Tf, are in arithmetical Progreſſion; $\square$ TC being the greateſt Term, and TC the Number of Terms; therefore $\odot$ TC $\times \frac{1}{3}$ TC $=$ the Sum of all the Series, by Lemma III.

Conſequently, $\odot$ AT $\times \frac{1}{2}$ TC $-$ $\odot$ TC $\times \frac{1}{3}$ TC $=$ the Sum of all the Series $\odot$ ab, $\odot$ ed, $\odot$ yf, &c. which conſtitute the Solidity of the Half-ſphere ATG. Put D $=$ 2 TC, the Axis of the Sphere; then $\frac{1}{3}$ D $=$ $\frac{2}{3}$ TC, and $\frac{1}{6}$ D $=$ $\frac{1}{3}$ TC. And becauſe $\square$ AT $=$ 2 $\square$ TC, therefore $\odot$ AT $=$ 2 $\odot$ TC $=$ 1.5708 DD; and 1.5708 DD $\times \frac{1}{4}$ D $=$ 0.3927 DDD.

Again; $\odot$ TC $\times$ TC $\frac{1}{3}$ $=$ 0.7854 DD $\times \frac{1}{6}$ D $=$ .1309 DDD, then 0.3927 DDD $-$ 0.1309 DDD $=$ 0.2618 DDD, the Solidity of the Half-ſphere.

Conſequently, 0.2618 DDD $\times$ 2 $=$ .5236 DDD will be the ſolid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whoſe Baſe, and Height, are each $=$ D.

For 0.7854 DDD $=$ the Solidity of the Cylinder, by Sect. V. But $\frac{2}{3}$ of 0.7854 DDD $=$ 0.5236 DDD, as before.

Scholium.

From this Demonſtration it will be eaſy to deduce, or raiſe Theorems for finding the ſolid Content of any Fruſtum of a Sphere; as a Tm, in the laſt Figure.

For we there ſuppoſe the Fruſtum aTm to be conſtituted of an infinite Series of Circles, which have the ſame Ratio with all thoſe Circles that conſtitute the Half-ſphere.

Therefore it follows, that $\odot$ aT $\times \frac{1}{2}$ Tb $-$ $\odot$ bT $\times \frac{1}{3}$ Tb, will be the Sum of all the Circles intercepted between T and b; conſequently it will be the Solidity of that Fruſtum.

And, becauſe $\square$ ab $+$ $\square$ Tb $=$ aT; therefore $\odot$ ab $+$ $\odot$ Tb $\times \frac{1}{2}$ Tb $-$ $\odot$ Tb $\times \frac{1}{3}$ Tb $=$ the Solidity. Let c $=$ ab half the Diameter of the Fruſtum's Baſe, h $=$ Tb its Height; and S $=$ the Solidity of the Fruſtum. Then $\odot$ ab $=$ 3.1416 cc, and $\odot$ Tb $=$ 3.1416 hh; conſequently,

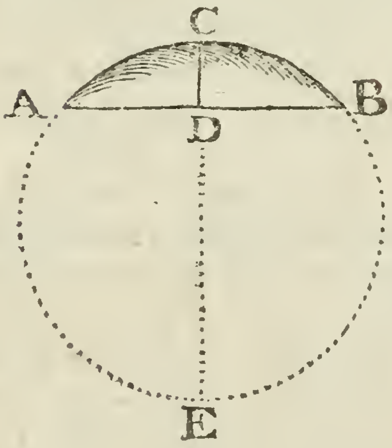
ſequently, $\frac{3.1416cch+3.1416hhh}{2}-\frac{3.1416hhh}{3}=S$.

Which, being reduced, will become $\overline{3cch+hhh}\times 0.5236=S$; which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words, thus:

If to three times the Square of the Semidiameter of the Fruſtum's Baſe, you add the Square of the Height of the Fruſtum, and multiply the Sum by the Height of the Fruſtum, and that Product multiplied by .5236, the Product will be the ſolid Content.

But if the Axis of the Sphere, and the Height of the Fruſtum, be given; then put $D=$ the Axis, $h=$ the Height of the Fruſtum, and c as before; it will be $\overline{D-h}\times h=cc$, *viz.* $Dh-hh=cc$. Then will $3Dhh-2hhh=3cch+hhh$; conſequently, $\overline{3Dhh-2hhh}\times 0.5236=S$, the Fruſtum's Solidity: Which is another Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words, thus:

From three times the Axis ſubtract twice the Height of the Fruſtum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this laſt Product will be the Solidity of the Fruſtum.



Example. Let ABCD be the Fruſtum of a Sphere; ſuppoſe AB (the Diameter of the Fruſtum's Baſe) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.

By

By the firſt Rule.

```
  8
  8
 ---
 64 Square of the Semidiameter AD.
  3
 ---
192
 16 add the Square of CD.
 ---
208
  4 multiply by CD.
 ---
832                  .5236
                      832
                   --------
                     10472
                    15708
                   41888
                  --------
                  435.6352
```

By the ſecond Rule, thus:

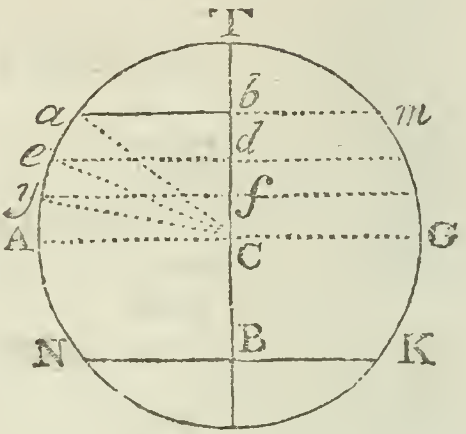
Firſt, by the Rule in Page 115, you will find the Axis of the whole Globe to be 20 Inches.

```
        20 Axis.           .5236
         3                  832
        ---              --------
From    60                 10472
Subtr.   8 twice CD.      15708
        ---              41888
Rem.    52               --------
Mult.   16 Sq. of CD.   435.6352 { the ſolid Content,
       ----                      { the ſame as before.
       312
       52
       ----
Prod.  832
```


And,

And, if it be required to find the middle Part, amNK, ufually called the middle Zone.

Then, becaufe it is fuppofed that am = NK, or (which is all one) that bC = CB; therefore it is plain, that if twice the Segment, aTm, be taken from the whole Sphere, there will remain the middle Zone am NK.



But becaufe the Work is a little troublefome, I will here fhew how to raife a Theorem for the doing it:

Firft, becaufe AC = yC = eC = aC = TC; therefore it will be □ AC — □ Cf = □ yf, □ AC — □ Cd = □ ed, □ AC — □ Cb = □ ab, *&c.*

Here becaufe □ AC, □ AC, □ AC, *&c.* are a Series of Equals, and Cb the Number of all the Terms; therefore □ AC × Cb = the Sum of all that Series (*per* Lemma I.)

And □ Cf, □ Cd, □ Cb, *&c.* being a Series of Squares, whofe Roots are in arithmetical Progreffion, beginning at the Center, C; *viz.* o, Cf, Cd, Cb, *&c.* wherein the greateft Term is □ Cb, and the Number of Terms is Cb; therefore □ Cb × $\frac{1}{3}$ Cb = the Sum of all the Series (*per* Lemma III.)

Confequently, the ⊙ AC × Cb — ⊙ Cb × $\frac{1}{3}$ Cb = the Sum of all the Series ⊙ yf, ⊙ ed, ⊙ ab, *&c.* which do conftitute the Solidity of the half Zone am AG.

And

And becauſe $\square AC - Cb = \square ab$; therefore $\odot AC - \odot ab = \odot Cb$. Conſequently $\odot AC \times Cb$ $\frac{\overline{\odot AC + \odot ab} \times Cb}{3} = 2\,\overline{\odot AC + \odot ab} \times \frac{1}{3} Cb$ will be the Solidity of the half Zone.

Put $D = AG = 2AC$, $x = am$, and $H = bB = 2Cb$.

Then $\odot AC = .7854DD$, $\odot ab = .7854xx$. And if we turn the common Factor .7854 into the Diviſor 1.27323, and then take the triple of that Diviſor; *viz.* 3.8197, the Reſult of the preceding Work will produce the following Theorem.

Theo. $\left\{ \frac{2DD + xx}{3.8197} : \times H = \right\}$ the middle Zone amNK.

Which in Words is thus: To twice the Square of the Axis AG, add the Square of the Diameter of the Fruſtum's Baſe (am), divide the Sum by 3.8197, then multiply the Quotient by the Height or Thickneſs of the middle Zone, and the Product will be the Solidity of the middle Zone required.

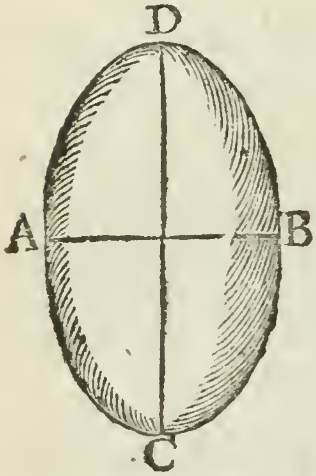
This is ſo plain and eaſy, that it needs no Example.

§ XII. *Of a* Spheroid.

A Spheroid is a Solid reſembling an Egg. To find the ſolid Content of it, this is

The RULE.

Multiply the Square of the Diameter of the greateſt Circle by the Length, and that Product multiply again by 5236; this laſt Product will be the Solidity of the Spheroid.



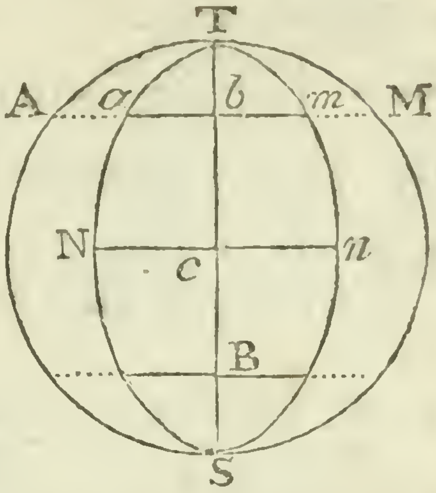
Let AB, the Diameter of the greateſt Circle, be 33 Inches, and CD (the Length) 55 Inches, the Solidity is required.

```
   33
   33
 ----
   99
  99
 ----
 1089
   55
 ----
 5445
5445
-----
59895
```

```
     59895
     .5236
    ------
    359370
   179685
  119790
 299475
----------
31361.0220 the Solidity.
```

Demon-

Demonſtration. Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheroid, and its Height equal to the Length of the Spheriod.



Suppoſe the Figure NTnSN, in the annexed Scheme, to repreſent a Spheroid, formed by the Rotation of the Semi-Ellipſis TNS, about its tranſverſe Axis TS.

Let D = T S, the Length of the Spheroid, and the Axis of the circumſcribing Sphere; and d = Nn, the Diameter of the greateſt Circle of the Spheroid:

Then, becauſe □ T C : □ N C : : □ A b : □ a b, by *Sect.* XV. *Step.* 3. *Page* 125.

Therefore it will be, D D : dd : : □ A b : □ a b.

But the Sum of an infinite Series of ſuch Circles as ⊙ Ab (whoſe Diameters are Chords) do conſtitute the Solidity of the Sphere. (By *Sect.* XI.)

And the Sum of an infinite Series of ſuch Circles as ⊙ a b (*viz.* whoſe Diameters are Ordinates of the Ellipſis) do conſtitute the Solidity of the Spheroid.

Therefore, D D : dd : : 0.5236 D D D : 0.5236 Ddd = the Solidity of the Spheroid. (*Eucl.* 5. 12)

But 0.5236 Ddd = $\frac{2}{3}$ of the Cylinder, whoſe Diameter is = d, and Height = D. (By *Sect.* V.)

Now, from this Proportion, between the Sphere and its inſcribed Spheroid, it will be very eaſy to deduce Theorems for finding the ſolid Content, either of the Fruſtum or middle Zone of any Spheroid; having the ſame Height with that of the Sphere; for,

As the Solidity of the whole Sphere is to the Solidity of the whole Spheroid, ſo is any Part of the Sphere to the like Part of the Spheroid.

As for Inſtance: Suppoſe it was required to find the middle Zone of any Spheroid.

Let D=TS, and d=Nn, as above; and H=bB, x=AM, and c=am.

Then $\left\{\frac{2DD+xx}{3.8197}\times H = \right.$ the middle Zone of the Sphere. And 0.5236 DDD : 0.5236 ddD :: $\frac{2DD+xx}{3.8197}\times H : \frac{2ddH}{3.8197} + \frac{xxddH}{3.8197DD} =$ the middle Zone of the Spheroid.

Again, DD : dd :: xx : cc. Therefore $\frac{xxdd}{DD} = cc$. Conſequently, $\frac{xxdd}{DD} \times \frac{H}{3.8197} = \frac{cc}{3.8197} \times H$: Which being taken inſtead of $\frac{xxddH}{3.8197DD}$, there will ariſe the following Theorem $\left\{\frac{2dd+cc}{3.8197} : \times H = \right.$ the middle Zone of the Spheroid.

Note, That 3.8197 = 1.2732 × 3. See *Page* 103.

§ XIII. *Of a Parabolic* Conoid.

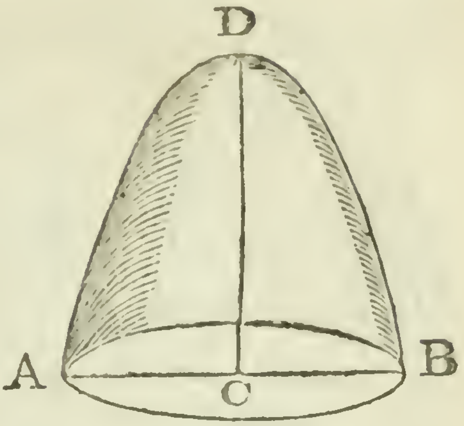
A Parabolic Conoid is ſomething like a half Spheroid, having its Sides ſomewhat ſtraiter. It is generated by ſuppoſing a Semi-parabola turned about its Axis. To find the ſolid Content of it, this is

The RULE.

Multiply the Square of the Diameter of its Baſe by .7854, and multiply that Product by half the Height, that laſt Product ſhall be the ſolid Content.

Let

Let A B C D be a Parabolic Conoid, the Diameter of its Baſe 36 Inches, and its Height, CD, 33 Inches; the Solidity is required.



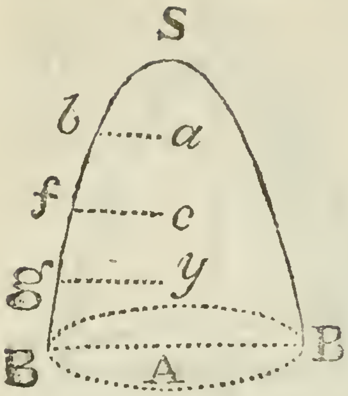
36	.7854	1017.8784
36	1296	33
216	47124	30536352
108	70686	30536352
	15708	
1296	7854	2)33589.9872
	1017.8784	16794.9936

1728)16794.9936(9.719 Feet, the Content.
15552
12429
12096
3339
1728
16113
15552
561

Demonſtration. The Parabolic Conoid is conſtituted of an infinite Number of Circles, whoſe Diameters are the Ordinates of the Parabola. Now, according to the Property of every Parabola, it will be,

$$SA : AB :: AB : \frac{\square AB}{SA} = L,$$ the *Latus Rectum*.

Then



Then $\begin{cases} Sa \times L = \square ba, \\ Se \times L = \square fe, \\ Sy \times L = \square gy, \text{ \&c.} \end{cases}$

Here $SA \times L$, $Se \times L$, $Sy \times L$, *&c.* are a Series of Terms in arithmet. Progreſſ. Therefore $\square$ b a, $\square$ f e, $\square$ g y, *&c.* are alſo a Series of Terms in the ſame Progreſſion, beginning at the Point S, wherein $\square$ AB is the greateſt Term, and S A, the Number of all the Terms. Therefore $\square\, AB \times \frac{1}{2} SA =$ the Sum of all the Series. (By Lemma II.)

Conſequently, $\odot\, AB \times \frac{1}{2} SA =$ the Sum of all the Series of $\odot$ ba, $\odot$ fe, $\odot$ gy, *&c.* which conſtitute the Solidity of the Conoid.

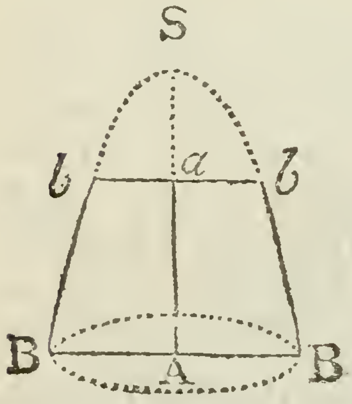
Put $D = 2AB$, and $H = SA$.

Then $.7854\, DD \times \frac{1}{2} H = .3927\, DDH$ will be the ſolid Content of the Conoid; which is juſt half the Cylinder, whoſe Baſe is $= D$, and Height $= H$.

This being rightly underſtood, it will be eaſy to raiſe a Theorem for finding the lower Fruſtum of any Parabolic Conoid.

For, ſuppoſing $h = aA$, the Height of the Fruſtum, and $p = Sa$, the Height of the Part bSb cut off, and $h + p = SA$, the Height of the whole Conoid,

Conſequently, $\dfrac{\odot AB \times H + \odot AB \times p}{2} =$ the Solidity of the whole Conoid.



And $\dfrac{\odot ba \times p}{2} =$ the Solidity of the Part cut off.

Therefore

Therefore	1	$\frac{\odot AB \times b + \odot AB \times p - \odot ba \times p}{2}$ is the Solidity of the Fruftum.
But	2	$b + p : \Box AB :: p : \Box ba$
Confeq.	3	$b + p : \odot AB :: p : \odot ba$
$3 \because$	4	$\odot AB \times p := \odot ba \times b + \odot ba \times p$
$4 - \odot ba \times p$	5	$\odot AB \times p \odot p ba \times = \odot ba \times b$
$\overline{}$ 1×2	6	$\odot AB \times b + \odot AB \times p - \odot ba \times p = 2F.$
$6 - 5$	7	$\odot AB \times b = 2F - \odot ba \times b.$
$7 + \odot ba \times h$	8	$\odot AB \times b + \odot ba \times b = 2F.$
$8 \div 2$	9	$\frac{\odot AB + \odot ba}{2} \times b = F$, the Fruftum's Solidity.

Let $D = 2\,AB$, as before, and $d = 2\,ba$, the Diameter of the Part cut off; then we fhall have the following Theorem.

$\overline{0.3927\,DD + 0.3927\,dd} : \times b =$ the Solidity of the Fruftum required: Which in Words is thus:

Multiply the Sum of the Squares of the greater and leffer Diameters by .3927, and the Product by the Height of the Fruftum, the laft Product fhall be the folid Content.

§ XIV. *Of a Parabolic* Spindle.

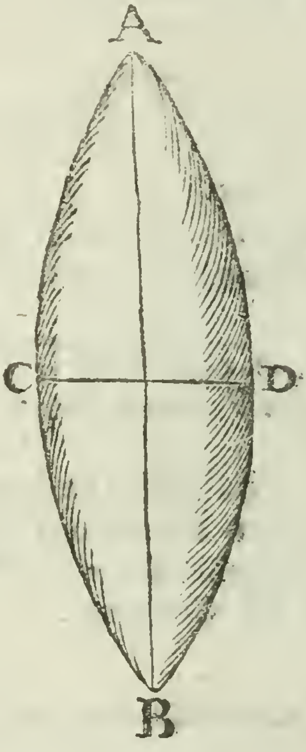
IF an acute Parabola be fuppofed to be moved about its greateft Ordinate, it will form a Solid, called a Parabolic Spindle. To find the folid Content, this is

The RULE.

Multiply the Square of the Diameter of its greateſt Circle by .41888 (being $\frac{8}{15}$ of .7854) and that Product by its Length; that laſt Product is the ſolid Content.

Let ABCD be a Parabolic Spindle, whoſe greateſt Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is required.

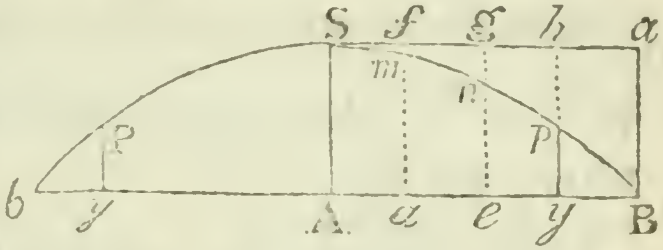
```
 36=CD          .41888
 36               1296
----            -------
 216            251328
108            376992
----           83776
1296 Square.  41888
              ----------
              542.86848
                     99
             -----------
              488581632
             488581632
             -----------
1728)53743.97952(31.10184
          . .....
        -----
        1903
         1759
          3179
           14515
            6912
           -----
           ....
```



The ſolid Content is 31.10184 Feet.

Demonftration. A Parabolic Spindle is conftituted of an infinite Series of Circles, whofe Diameters are all parallel to the Axis of the Parabola, as ⊙ m a, ⊙ n e, ⊙ p y, *&c.*

Let us fuppofe the Line S d parallel to A B, *&c.* Then it hath already been proved, that the Lines f m, g n, h p, *&c.* are a Series of Squares, whofe Roots are in arithmetical Progreffion, confequently their Squares, *viz.* □ f m, □ g n, □ h p, *&c.* will be a Series of Biquadrates, whofe Roots will be in arithmetical Progreffion: Which being premifed, we may proceed thus:



First	1	S A — f m = m a.
	2	S A — g n = n e.
	3	S A — h p = p y.
1 ⊜ 2	4	□ S A — 2 S A × f m + □ f m = □ m a.
2 ⊜ 2	5	□ S A — 2 S A × g n + □ g n = □ n e.
3 ⊜ 2	6	□ S A — 2 S A × h p + □ h p = □ p y, *&c.*

1. In thefe Equations, the □ S A, □ S A, □ S A, being a Series of Equals, and A B the Number of all the Terms; therefore it will be □ S A × A B = the Sum of the Series. By Lemma I.

2. Becaufe f m, g n, h p, *&c.* are a Series of Squares, wherein S A is the greateft Term, and A B the Number of all the Terms.

Therefore $\frac{2\,SA \times SA \times AB}{3} = \frac{2\,\square SA \times AB}{3}$ will be the Sum of the Series. (By Lemma III.)

3. And

3. And the $\square$ fm, $\square$ gn, $\square$ hp, &c. will be a Series of Terms in the Ratio of Biquadrates, as above; $\square$ SA being the greateft Term, and AB the Number of all the Terms. Therefore it will be $\frac{\square SA \times AB}{5}$ = the Sum of all the Series. (By Lemma V.)

Whence it follows that $\square SA \times AB - \frac{2 \square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the Sum of all the Series of $\square$ ma, $\square$ ne, $\square$ py, &c.

That is, $\frac{8 \square SA \times AB}{15}$ = the Sum of all the Series $\square$ ma, $\square$ ne, $\square$ py, &c. Confequently, $\frac{8 \odot SA \times AB}{15}$ = the Sum of all the Series of Circles, $\odot$ ma, $\odot$ ne, $\odot$ py, &c. which conftitute the Solidity of half the Spindle; *viz.* of SAB.

Therefore putting D=2SA, and H=2AB, it will be 0.41888 DDH= the Solidity of the whole Parabolic Spindle bSB, being $\frac{8}{15}$ of 0.7854 DDH, the Solidity of its circumfcribing Cylinder.

From hence we may alfo raife a Theorem for finding the Fruftum, SA py, of the laft Figure.

For $\odot$ SA being the greateft Term, $\odot$ py the leaft Term, and Ay the Number of all the Terms or Circles included between A and y:

Therefore,

Therefore	1	$\overline{\square SA - 2SA \times hp + \frac{\square hp}{5}} \times Ay =$ z the Sum of all the Series, $\square SA$, $\square ma$, $\square en$, $\square py$.
1×3	2	$\overline{3\square SA - 2SA \times hp + \frac{3\square hp}{5}} \times Ay = 3z$
$z \div Ay$	3	$3\square SA - 2SA \times hp + \frac{3\square hp}{5} = \frac{3z}{Ay}$
But	4	$\square SA - 2SA \times hp = \square py - \square hp$, *per St.* 6.
$3 - 4$	5	$2\square SA + \frac{3\square hp}{5} = \frac{3z}{Ay} - \square py + \square hp$.
$5 +$ &c.	6	$2\square SA + \square py - \frac{2}{5} \square hp = \frac{3z}{Ay}$
Confeq.	7	$\overline{2\odot SA + \odot py - \frac{2}{5}\odot hp} : \times \frac{1}{3} Ay = z$,

the Sum of all the Series of $\odot SA$, $\odot ma$, $\odot ne$, $\odot py$; which conſtitute the Solidity of the Fruſtum S A p y. Therefore putting $D = 2SA$, as before, $C = 2py$, $x = 2hp$, and $H = Ay$; it will be $\overline{1.5708\,DD + .7854\,CC - xx} : \times \frac{1}{3} H =$ the Fruſtum S A p y. And if we make $L = 2H$, then $\overline{1.5708\,DD + .7854\,CC - .31416\,xx} : \times \frac{1}{3} L =$ the Double of that Fruſtum, being the middle Zone. Which in Words is thus:

Multiply the Square of the greateſt Diameter by 1.5708, and multiply the Square of the leſſer Diameter by .7854, and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products ſubtract the latter Product, and multiply the Remainder by one-third Part of the Length, and that Product will be the Solidity of the middle Zone required.

CHAP. III.

Of the Meaſuring the Works of the ſeveral Artificers relating to Building; and what Methods and Cuſtoms are obſerved in doing it.

§ I. *Of* Carpenters *Work.*

THE Carpenters Works, which are meaſurable, are Flooring, Partitioning, and Roofing; all which are meaſured by the Square of 10 Feet long, and 10 Feet broad; ſo that one Square contains 100 Square Feet.

1. *Of Flooring.*

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad; How many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, &c. which divide by 100 (this is done by cutting from the Product two Figures towards the Right-hand, with a Daſh of the Pen); and the remaining Figures are the Quotient, and

and the Figures cut off are Feet: Thus, 1631 divided by 100, by cutting off 31 from the Right-hand of it, the Quotient is 16 Squares, and the 31 cut off is 31 Feet.

See the Work both by Decimals, and alſo by Feet and Inches.

		F.	I.	
57.25		57	3	
28.5		28	6	
28625		456		
45800		114		
11450		28	7	6
16|31.625		7	0	0
		16|31	7	6

Facit 16 Squares and 31 Feet.

Note, That 5 is the Decimal for half of any thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; ſo in the laſt Example, .25 is the Decimal of 3 Inches, becauſe 3 Inches is a Quarter of a Foot; and 5 is the Decimal of 6 Inches, becauſe 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad; How many Squares are contained in that Floor?

		F.	I.	
47.75		53	6	
53.5		47	9	
23875		371		
14325		212		
23875		26	9	
25|54.625		13	4	6
		23	6	
		25|54	7	6

Facit 25 Squares and 54 Feet.

By Scale and Compaſſes.

In the firſt Example, extend the Compaſſes from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the ſecond Example, extend the Compaſſes from 1 to 47.5, that Extent will reach from 53.5 to 25 Squares and above a Half.

1. *Of Partitioning.*

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1010.625; which divided by 100 (as before is ſhewed) and the Anſwer is 10 Squares 10 Feet; the Inches or Parts, in theſe Caſes, are of no Value.

12.25		F.	I.	
82.5		82	6	
		12	3	
6125				
2450		990	0	
9800		20	7	6
10|10.625		10|10	7	6

Facit 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and its Breadth 11 Feet 3 Inches; How many Squares are contained in it?

The Length and Breadth being multiplied together, the Product is 1032 Feet; which divided by 100, the Anſwer will be 10 Squares and 32 Feet.

91.75	F.	I.		
11.25	91	9		
45875	11	3		
18350	1009	3		
9175	22	11	3	
9175	10|32	2	3	
10|32.1875				

3. *Of Roofing.*

It is a Rule amongſt Workmen, that the Flat of any Houſe, and half the Flat thereof, taken within the Walls, is equal to the Meaſure of the Roof of the ſame Houſe; but this is when the Roof is true pitched: For if the Roof be more flat or ſteep than the true Pitch, it will meaſure to more or leſs accordingly.

Example 1. If a Houſe within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad; How many Squares of Roofing will cover that Houſe?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half of this is 406 Feet; which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Anſwer is 12 Squares and 18 Feet.

	18.25
	44.5
	9125
	7300
	7300
Flat	812.125
Half	406
	12|18

	F.	I.	
	44	6	
	18	3	
	352		
	44		
	11	1	6
	9	0	0
The Flat	812	1	6
The Half	406		
Sum	12|18		

Facit 12 Squares 18 Feet.

By Scale and Compaſſes.

In the firſt Example of Partitioning, extend the Compaſſes from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and One Tenth.

In the ſecond Example, extend the Compaſſes from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little leſs than a third Part.

In the Example of Roofing extend the Compaſſes from 1 to 18.25, that Extent will reach from 44.5 to 812, the Flat; to which add the Half thereof, and the Sum is 12.18; which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, which are meaſured by the Foot, running Meaſure, that is, by the Number of Feet in Length only; as Cornices, Doors and Caſes, Window-frames, Guttering, Lintels, Sommers, Skirt-boards, &c.

Note 1. In the Meaſuring of Flooring, after you have meaſured the whole Floor, you muſt deduct out of it the Well-holes for the Stairs and Chimnies; and in Partitioning, for the Doors, Windows, &c. except (by Agreement) they are to be included.

Note

Note 2. In meaſuring of Roofing, ſeldom any Reductions are made for the Holes for the Chimney-ſhafts, the Vacancies for Lutheren-lights and Sky-lights; for they are more Trouble to the Workman than the Stuff which would cover them is worth.

§ II. *Of* BRICKLAYERS *Work.*

THE principal is Tiling, Walling, and Chimney-work.

1. *Of Tiling.*

Tiling is meaſured by the Square of 100 Feet, as Flooring, Partitioning, and Roofing were in the Carpenters Work; ſo that between the Roofing and Tileing, the Difference will not be much; yet the Tiling will be the moſt; for the Bricklayers ſometimes will require to have double Meaſure for Hips and Vallies. When Gutters are allowed double Meaſure, the Way is to meaſure the Length along the Ridge-tile, and by that Means the Meaſure of the Gutters becomes double; it is uſual alſo to allow double Meaſure at the Eaves, ſo much as the Projector is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contained therein?

F. I,

F.	I.	
37	3	
45	0	
185		
148		
11	3	
16|76	3	

37.25
45
18625
14900
16|76.25

Anſwer, 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 Inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tileing are in the Roof?

F.	I.	
43	6	
35	9	
215		
129		
21	9	
10	10	6
17	6	
15|55	1	6

37.75
43.5
17875
10725
14300
15|55.125

Here the Length and Depth being multiplied together, the Product is 1555 Feet; which divided by 100 (as before is taught) the Anſwer is 15 Squares and 55 Feet.

By Scale and Compaſſes.

In the firſt Example, extend the Compaſſes from 1 to 37.25, that Extent will reach from 45 to 16 Squares, and a little above three Quarters of a Square.

In

In the ſecond Example, extend the Compaſſes from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet; that is, a little above a Half-ſquare.

2. *Of Walling.*

Bricklayers commonly meaſure their Work by the Rod Square of 16 Feet and a half; ſo that one Rod in Length, and one in Breadth, contain 272.25 Square Feet; for 16.5, multiplied into itſelf, produces 272.25 Square Feet. But in ſome Places the Cuſtom is to allow 18 Feet to the Rod; that is, 324 Square Feet. And in ſome Places the uſual Way is, to meaſure by the Rod of 21 Feet long and 3 Feet high, that is, 63 Square Feet; and here they never regard the Thickneſs of the Wall, but the uſual Way is to moderate the Price according to the Thickneſs.

When you meaſure a Piece of Brick-work, the firſt thing is to enquire by which of thoſe Ways it muſt be meaſured; then, having multiplied the Length and Breadth in Feet together, divide the Product by the proper Diviſor, either for Rods or Roods, and the Quotient is Square Rods, or Square Roods, accordingly.

But commonly Brick-walls, that are meaſured by the Rod, are to be reduced to a Standard-thickneſs; *viz.* of a Brick and a half thick (if it be not agreed on the contrary); and to reduce a Wall to Standard-thickneſs, this is

The RULE.

Multiply the Number of ſuperficial Feet that are found to be contained in any Wall by the Number of Half-bricks which that Wall is in Thickneſs; one third Part of that Product ſhall be the Content in Feet, reduced to the Standard-thickneſs of one Brick and a half.

Example.

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick; How many Rods of Brick-work are contained therein, when reduced to the Standard?

```
                 19.25 Height.
                  72.5 Length.
                ------
                  9625
                 3850
               13475
             --------
              1395.625
                    11
          ------------
           3)15351.875
          ------------
272.25)  5117.291(18 Rods.
              --------
                239479
                ------
       68.06)21679(3 Quarters of a Rod.
                ------
                 12.61
```

Anfwer, 18 Rods 3 Quarters 12 Feet.

F.	I.	
72	6	
19	3	
648		
72		
18	1	6
9	6	0
1395	7	6
		11
3)15351	10	6

272)5117(18 Rods.

2397

68)221(3 Quarters of a Rod.

17

Note, That 68.06 is one-fourth Part of 272.25.

Note also, That in reducing of Feet into Rods, they usually reject the odd Parts, and divide only by 272, as is done in the second Way of the last Example; so the Answer, by the second Way, is 18 Rods 3 Quarters and 17 Feet; more by about $4\frac{1}{2}$ Feet than by the first Way, where it is done decimally; a thing very insignificant.

Example

Example 2. If a Wall be 245 Feet 9 Inches long, 16 Feet 6 Inches high, and two Bricks and a half thick; I demand how many Rods of Brick-work are contained in it, when reduced to Standard-Thicknefs?

```
      245.75
        16.5
      ------
      122875
     147450
     24575
    --------
    4054.875
         5
   -------
  3)20270
   -------
272)6756(24 Rods.
     ----
     1316
     ----
  68)228(3 Quarters of a Rod.
      ---
       24
```

Anfwer, 24 Rods 3 Quarters 24 Feet.

```
      F.   I.
     245   9
      16   6
    ---------
    1470
    245
     122  10  6
      12   0  0
    ------------
    4054  10  6 Anfwer in Feet.
```

Before

Before I ſhew how to work the two laſt Examples by Scale and Compaſſes, I will ſhew how to find proper Diviſors to facilitate the Operation; becauſe it would be too intricate and tedious to perform by Scale and Compaſſes, according to the Rule above taught.

To find proper Diviſors.

Divide 3 (the Number of Half-bricks in $1\frac{1}{2}$), by the Number of Half-bricks in the Thickneſs, the Quotient will be a Diviſor, which will give the Anſwer in Feet. But if you would have a Diviſor to bring the Anſwer in Rods at once, then multiply 272.25 by the Diviſor found for Feet, and the Product will be a Diviſor, which will give the Anſwer in Rods.

Example. Let it be required to find a Diviſor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6 (the Half-bricks in the Thickneſs) and the Quotient is .5, which is a Diviſor that will give the Anſwer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Diviſor, which will give the Anſwer in Rods; that is, as 136.125 is to the Length of the Wall, ſo is the Height to the Content in Rods. Or, as .5 is to the Length, ſo is the Height to the Content in Feet.

After the ſame Manner you may find Diviſors for any other Thickneſs, which you will find to be as expreſſed in the following little Table.

The Thicknefs of the Wall.	Divifors for the Anfwer in Feet.	Divifors for bringing the Anfwer in Rods.
1 Brick thick	1.5	408.375
$1\frac{1}{2}$ Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
$2\frac{1}{2}$ Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
$3\frac{1}{2}$ Bricks thick	.4285	116.659
4 Bricks thick	.375	102.0937

Let the fecond Example, aforegoing, be wrought by Scale and Compaffes, where the Length is 245.75, the Height 16.5, and the Thicknefs $2\frac{1}{2}$ Bricks.

Extend the Compaffes from 163.35 (the tabular Number againft $2\frac{1}{2}$ Bricks,) to 245.75; that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at $3\frac{1}{2}$ Bricks thick; How many Rods are contained therein?

Extend the Compaffes from 116.659 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13, that is, 12 Rods and a little above half a Quarter.

It will be very proper and commodious, for fuch as have frequent Occafion to meafure Brick-work, to have in the Line of Numbers little Brafs Center-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thicknefs of the Wall.

If

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high; How many Rods are contained in it?

104.75	F. I.
17.25	104 9
	17 3
52375	
20950	728
73325	104
10475	26 2 3
	12 9 0
63)1806.9375(28	
126	1806 11 3
546	
504	
42	

Answer, 28 Rods 42 Feet.

Note, That such as dig Cellars, frequently make them by the Floor, 18 Feet square, and a Foot deep, being a Floor of Earth; that is, 324 solid Feet.

3. *Of Chimnies.*

If you are to measure a Chimney standing alone by itself, without any Party-wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickness must be the same as the Jambs are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, because of the Gatherings of the Breast and Wings, to make room for the Hearth in the Story.

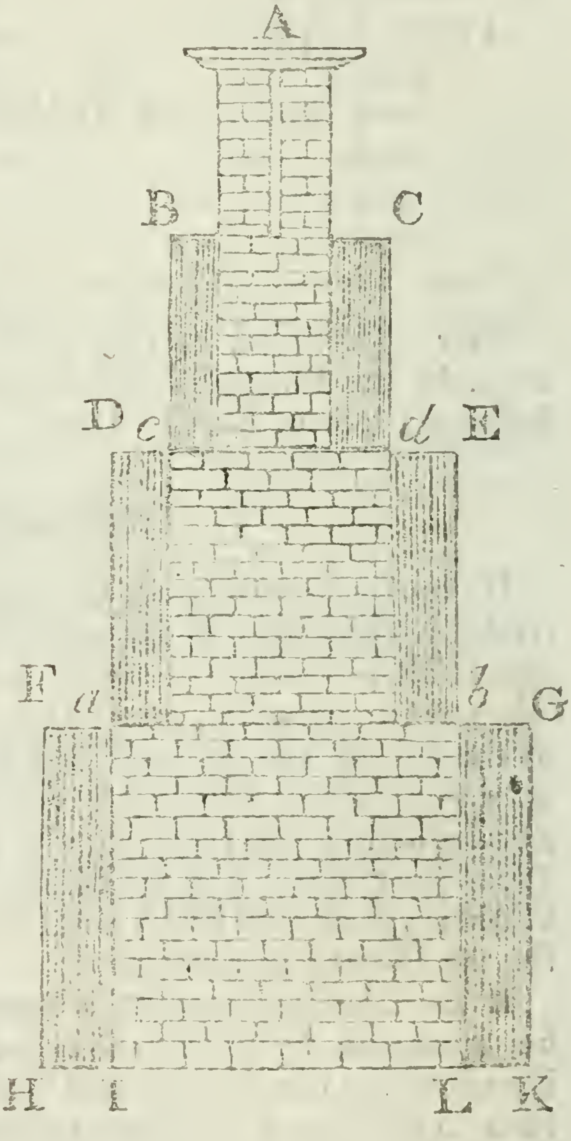
If the Chimney-back be a Party-wall, and the Wall be measured by itself, then you must measure the Depth of the two Jambs, and the Length of the Breast for a Length, and the Height of the Story the Breadth, at the same Thickness your Jambs were of.

When you measure Chimney-shafts, girt them with a Line round about the least Place of them, for

the Length, and the Height fhall be the Breadth: And if they be four Inch-work, then you muft fet down their Thicknefs at one Brick-work; but if they be wrought 9 Inches thick (as fometimes they are, when they ftand high and alone above the Roof,) then you muft account your Thicknefs $1\frac{1}{2}$ Brick, in Confideration of Widths and Pargetting, and Trouble in Scaffolding.

It is cuftomary, in moft Places, to allow double Meafure for Chimnies.

Example. Suppofe this Figure, ABCDEFGHIK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be meafured according to double Meafure.



Firft, I begin with the Breaft-wall IL, and the two Angles LK and HI, which together are 18 Feet 9 Inches; then take the Height of the Square HF, 12 Feet 6 Inches; which multiplied together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.

For

For the Square DaEb, the Length of the Breast-wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet; which multiplied together, make 130 Feet 6 Inches, for the Content of the Part DaEb.

Then the Height of the next Square 7 Feet, and the Length of the Breast-wall and two Angles is 10 Feet 3 Inches; which multiplied together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The Compass of the Chimney-shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches; which multiplied together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideness 1 Foot 3 Inches; which multiplied together, make 15 Feet, for the Content.

The Work.

	F.	I.			
	18	9			18.75
	12	6			12.5
					9375
	225	0			3750
	9	4	6		1875
FGHK	234	4	6	FGHK	234.375
	F.	I.			
	14	6			14.5
	9	0			9
DaEb	130	6		DaEb	130.5
	F.	I.			
	10	3			10.25
	7	0			7
BcCd	71	9		BcCd	71.75

F.	I.	
13	9	
82	6	
6	10	6
The Shaft 89	4	6

13.75
6.5
6875
8250
The Shaft 89.375

F.	I.
1	3
12	0
The Fetter 15	0

1.25
12
The Fetter 15.00

272)1082(3 Rods.

68) 266(3 Quarters.

Rem. 62 Feet.

	F.	I.	P.
FGHK	234	4	6
DaEb	130	6	0
BcCd	71	9	0
The Shaft	89	4	6
The Fetter	15	0	0
The Sum	541	0	0
The Double	1082	0	0

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or cuſtomary Meaſure; which Feet muſt be reduced to Rods, as was ſhewed before.

So the Feet in the foregoing Example being reduced to Rods (the Thickneſs being ſuppoſed $1\frac{1}{2}$ Brick) it makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rods wanting 6 Feet.

This is all he Meaſure that can be allowed, when the Chimney ſtands in a Gavel or Side-wall; in which Caſe the Back of the Chimney (here not meaſured) is accounted as Part of the Gavel; but if the Chimnies ſtand by themſelves, as all Stacks of Chimnies

in great Buildings do, in ſuch Caſe, it is all Chimney-work, and therefore ought to be meaſured double on all Sides.

§ III. *Of* PLASTERERS *Work*.

THE Plaſterers Works are principally of two Kinds; namely, 1. Works lathed and plaſtered, which they call Cieling. 2. Works rendered; which are of two Kinds; *viz.* upon Brick-walls, or between Quarters, in the Partitions between Rooms: All which are meaſured by the Yard-ſquare, or Square of 3 Feet, which is 9 Feet.

1. *Of Cieling.*

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad; How many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

F.	I.	
59	9	
24	6	
236		
118		
29	10	6
18	0	0
1463	10	6

	59.75
	24.5
	29875
	23900
	11950
	9)1463.875
Anſwer	162.65

By

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 162.5 Yards.

2. *Of Rendering.*

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high; How many Yards are in thoſe Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Anſwer.

	F.	I.			
	141	6			141.5
	11	3			11.25
	1556	6			7075
	35	4	6		2830
					1415
	9)1591	10	6		1415
Anſwer	176	7			9)1591.875
					176.87

Anſwer, 176.87 Yards.

Extend the Compaſſes from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

Note 1. If there be any Doors, Windows, or the like, in your Partitioning, you muſt make Deductions for them.

Note 2. When you meaſure Rendering upon Brick-walls, you are to make no Deductions; but when you meaſure Rendering between Quarters, you may very well deduct one fifth Part for the Quarters, Braces, and Interſtices.

Note

Note 3. That Whiting and Colouring are both meaſured by the Yard, as Cieling and Rendering were; and, as in Rendering between Quarters, you deduct one fifth Part, ſo in Whiting and Colouring you muſt add one fourth or one fifth Part at leaſt, for the Projections of the Quartering, &c.

§ IV. *Of* JOYNERS *Work.*

JOYNERS meaſure their Work by the Yard-ſquare; but in taking their Dimenſions, they differ from ſome others; for they have a Cuſtom, and ſay, *We ought to meaſure where our Plane touches:* Wherefore in taking the Height of any Room, where there is a Cornice about, and ſwelling Panels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings; which will make the Room to meaſure much higher than it is: Then for meaſuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room or Wainſcot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet and 3 Inches in Compaſs: How many Yards doth that Room contain?

Multiply the Compaſs by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Anſwer.

F.	I.	
126	3	
15	9	
630		
126		
63	1	6
31	6	9
3	9	0
9)1988	5	3
Anfwer, 220	8	

126.25
15.75
63125
88375
63125
12625
9)1988.4375
220.8

Facit 220 Yards 8 Feet.

Example 2. If a Room of Wainfcot be 16 Feet 3 Inches high (being girt over the Mouldings), and the Compafs of the Room 137 Feet 6 Inches; How many Yards are contained in it?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

F.	I.	
137	6	
16	3	
830		
137		
34	4	6
9)2234	4	6
248	2	0

137.5
16.25
6875
2750
8250
1375
9)2234.375
248.2

Facit 248 Yards 2 Feet.

By

By Scale and Compasses.

For the first Example, extend the Compasses from 9 to 126.25, that Extent will reach from 15.75 to 220.9 Yards.

For the second Example, extend the Compasses from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards and above a Quarter.

In Joyners Work there is another Thing to be observed; that is, in the measuring of Doors, Window-shutters, and all such Work as is wrought on both Sides, they are paid for Work and Half-work; so that in measuring all such Work, you must first find the Content, as before, and take half that Content, and add to it; so shall the Sum be the Content at Work and half.

Example. If the Window-shutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high; How many Yards are contained therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the Half of which is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 6 Feet nearly, the Content at Work and half.

F.	I.		
69	9		69.75
6	3		6.25
418	6		34875
17	5	3	13950
			41850
435	11	3	
217	11	7	435.9375
			217.9687
9)653	10	10	
			653.9062
72	6 nearly.		

Facit 248 Yards 6 Feet, nearly.

By

By Scale and Compaffes.

Extend the Compaffes from 9 to 69.75, that Extent will reach from 6.25 to 48.4 Yards; the Half of which is 24.02; and thefe added together, make 72.6 Yards, the Content at Work and half.

Note, That you muft make Deductions for all Window-lights; but you muft meafure the Window-boards, Sopheta-boards, and Cheeks, by themfelves.

§ V. *Of* PAINTERS *Work.*

THE taking the Dimenfions of Painters Work is the fame as that of Joyners, by girting over the Mouldings and fwelling Panels, in taking the Height; and it is but Reafon that they fhould be paid for that on which their Time and Colour are both expended. The Dimenfions thus taken, the cafting up, and reducing Feet into Yards, is altogether the fame as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice coloured over. Only take Notice, that Window-lights, Window-bars, Cafements, and fuch-like Things, they do at fo much a Piece.

Example. If a Room be painted, whofe Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compafs of the Room 97 Feet 9 Inches; How many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 10 Inches 6 Parts; which being divided by 9, the Quotient is 179 Yards and 2 Feet, nearly.

F. I.	
97 9	97.75
16 6	16.5
584	48875
98	58650
48 10 6	9775
9)1612 10 6	9)1612.875
179 1	179.2

Facit 179 Yards 2 Feet, nearly.

By Scale and Compasses.

Extend the Compasses from 9 to 16.5, that Extent will reach from 97.75 to 179.2 Yards.

§ IV. *Of* GLASIERS *Work.*

GLASIERS measure their Work by the Foot square; so that the Length and Breadth of a Pane of Glass in Feet, being multiplied into each other, produceth the Content.

Note, That Glasiers usually take their Dimensions to a Quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part subdivided into 12, *&c.*

Example. If a Pane of Glass be 4 Feet 8 Inches and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter broad; How many Feet of Glass are in that Pane?

The Decimal of $\left\{ \begin{array}{l} \text{8 Inches } \frac{3}{4} \\ \text{4 Inches } \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .729 \\ .354 \end{array} \right.$

F.	I.	P.			
4	8	9			4729
1	4	3			1.354
					18916
4	8	9			23645
1	6	11	0		14187
	1	2	2	3	4729
6	4	10	2	3	6.403066

Anfwer, 6 Feet 4 Inches.

By Scale and Compaffes.

Extend the Compaffes from 1 to 1.345, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be 8 Panes of Glafs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad; How many Feet of Glafs are contained in the faid 8 Panes?

The Decimal of $\left\{\begin{matrix}7 \text{ Inches } \frac{3}{4} \\ 5 \text{ Inches } \frac{1}{4}\end{matrix}\right\}$ is $\left\{\begin{matrix}.646 \\ .437\end{matrix}\right.$

F.	I.	P.			
4	7	9			4.646
1	5	3			1.4 7
					32522
4	7	9			13938
1	11	2	9		18584
	1	1	11	3	4646
6	8	1	8	3	6.676302
				8	8
53	5	1	6	0	53.410416

Facit 53 Feet 5 Inches.

By

By Scale and Compasses.

Extend the Compaſſes from 1 to 1.437, that Extent will reach from 4.646 to 6 676; then extend the Compaſſes from 1 to 8, that Extent will reach from 6.676 to 53.4, the Content.

Example 3. If there be 16 Panes of Glaſs, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad; How many Feet of Glaſs are contained in them?

F.	I.	P.			
4	5	6			4.458
1	4	9			1.395
4	5	6			22290
1	5	10	0		40122
	3	4	1	6	13374
					4458
6	2	8	1	6	6.218910
				4	4
24	10	8	6	0	24.875640
				4	4
99	6	10	0	0	99.502560

Facit 99 Feet 6 Inches.

Note, That inſtead of multiplying by 16, I have multiplied by 4 twice, becauſe 4 times 4 is 16.

By Scale and Compasses.

Extend the Compaſſes from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compaſſes from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

Note, That when Windows have Half-rounds at the Top, they meaſure them at the full Height, as if they were ſquare. Alſo round or oval Windows are meaſured

ſured at the full Length and Breadth of their Diameters. Likewiſe Crotchet-windows in Stone-work are all meaſured by their full Squares. And there is Reaſon for ſo doing; for the Trouble in taking their Dimenſions to work by, the Waſte of Glaſs in working, and the Time expended in ſetting up, is far more than the Glaſs can be valued at.

§ V. *Of* MASONS *Work.*

MASONS meaſure their Work ſometimes by the Foot ſolid, ſometimes by the Foot ſuperficial, and in ſome Places they meaſure their Walling by the Rood, that is, 21 Feet long and 3 Feet high, which is 63 ſquare Feet. Examples of each are as follow.

Example. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick; How many ſolid Feet are contained in that Wall?

F.	I.		
97	5		
18	3		
776			
97			
24	4	3	
6	0	0	
1	6	0	
1777	10	3	
2	3		
3555	8	6	
444	5	6	9
4000	2	0	9

97.417
18.25
487085
194834
779336
97417
1777.86025
2.25
888930125
355572050
355572050
4000.1855625

Multiply the Length, Height, and Thickneſs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contained in the Wall.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high; How many Feet ſuperficial are contained therein?

F.	I.			
107	9			107.75
20	6			20.5
2155	0			53875
53	10	6		215500
2208	10	6		2208.875

Facit 2208 Feet 10 Inches.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 107.75, that Extent will reach from 20.5 to 2208.875, the ſuperficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high; How many Roods are contained therein?

F.	I.	
112	3	
16	6	
676	0	
112		
56	1	6
1852	1	6

112.25
16.5
56125
67350
11225
63)1852.125(29
592
25

Facit 29 Roods 25 Feet.

By Scale and Compaffes.

Extend the Compaffes from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.

[CHAP.

CHAP. IV.

The Meaſuring of BOARD *and* TIMBER.

§ I. *Of* BOARD MEASURE.

TO meaſure a Board, is only to meaſure a long Square.

Example 1. If a Board be 16 Inches broad, and 13 Feet long; How many Feet are contained in it?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is a third Part of 144, the ſame as before.

$$\begin{array}{r} 12 : 13 :: 16 \\ 13 \\ \hline 48 \\ 16 \\ \hline 12)208 \\ \hline 17\tfrac{4}{12} \end{array}$$

$$\begin{array}{r} \text{Or, } 144 : 156 :: 16 \\ 16 \\ \hline 936 \\ 156 \\ \hline 144)2496(17\tfrac{48}{144} \\ 1056 \\ \hline 48 \end{array}$$

By Scale and Compaffes.

Extend the Compaffes from 12 to 13, that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Or, extend from 144 to 156 (the Length in Inches) that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad; How many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and fo many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch. Inch. Inch. Inch.
19 : 144 :: 1 : 7.58 *ferè.*

Extend

Extend the Compasses from 19 to 144, that Extent will reach from 1 to 7.58; that is, 7 Inches, and something more than a half. So, if a Board be 19 Inches broad, if you take 7 Inches, and a little more than a half with your Compasses from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains; or you may cut off from that Board any Number of Feet desired.

For this Purpose there is a Line upon most ordinary Joint-rules, with a little Table placed upon the End of all such Numbers as exceed the Length of the Rule, as in this little Table annexed.

0	0	0	0	5	0	$8\frac{1}{2}$	0
12	6	4	3	2	2	1	1
1	2	3	4	5	6	7	8

Here you see, if the Breadth be one Inch, the Length must be 12 Feet; if two Inches, the Length is 6 Feet; if five Inches broad, the Length is 2 Feet 5 Inches, &c.

The rest of the Lengths are expressed in the Line, thus: If the Breadth be 9 Inches, you will find it against 15 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, &c.

§ II. *Of* SQUARED TIMBER.

BY Squared Timber is here meant such Pieces of Timber as have equal Bases, and the Sides strait and parallel. The Rules for measuring all such Solids are shewed in § II. of Chap. II. to which I refer.

Example

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) ſquare, and 18 Feet long; How many ſolid Feet are contained in it?

```
     15
     15
     --
     75
    15
   ----
    225
     18
   ----
   1800
   225
   ----
144)4050(28.125
   ----
   1170
   ----
   180
    360
     720
     ---
       0
```

F.	I.	
1	3	
1	3	
1	3	
	3	9
1	6	9
		6
9	4	6
		3
28	1	6

Anſwer, 28 Feet and half a Quarter.

Here, inſtead of multiplying by 18, where I wrought by Feet and Inches, I multiplied by 6, and then by 3, becauſe 3 times 6 is 18.

Example 2. If a Piece of ſquared Timber be 2 Feet 9 Inches deep, 1 Foot 7 Inches broad, and 16 Feet 9 Inches long; How many Feet of Timber are in that Piece?

Multiply the Depth, Breadth, and Length together, and the Product will be the Content.

	33		
	19		
	297		
	33		
	627		
	1675		
	3135		
	4389		
	3762		
	627		
	144)10502.25(72.93		
	422		
	1342		
	465		
	33		

F.	I.		
2	9		
1	7		
2	9		
1	7	3	
4	4	3	
16	8	0	
69	9	0	
3	3	2	3
72	11	2	3

Anſwer, 72 Feet 11 Inches; or 72 Feet 93 Parts.

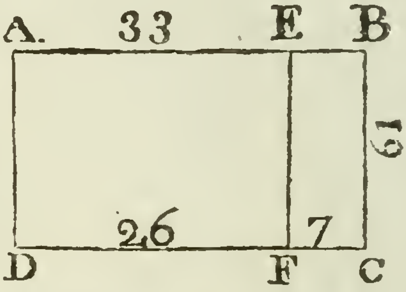
By Scale and Compaſſes.

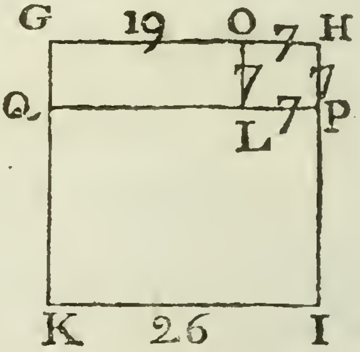
For the firſt Example, extend the Compaſſes from 12 to 15 Inches (the Side of the Square, that Extent will reach from 18 Feet (the Length being twice turned over) to 28 Feet, and ſomething more.

For the ſecond Example, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts, and the Compaſs Point will reſt upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compaſſes from 12 to 25 (the Proportional found) that Extent will reach (being twice turned over) from 16.75 Feet, the Length, to 72.93 Feet, the Content.

A common Error is committed, for want of Art, in meaſuring theſe laſt Sorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, though it be but ſmall, when the Depth and Breadth are pretty near equal; yet, if the Difference be great, the Error is very conſiderable; for the Piece of Timber thus meaſured, will be more than the Truth, by a Piece whoſe Length is equal to the Length of a Piece of Timber to be meaſured, and the Square equal to half the Difference of the Breadth and Depth, as I ſhall here demonſtrate.





I ſay, the Square GHIK is greater than the Parallelogram A B C D, by the little Square O H P L; for the Parallelogram QPIK is equal to the Parallelogram AEFD; and the Parallelogram G O L Q is equal to the Parallelogram E B C F. Therefore the Square is greater than the Parallelogram by the little Square O H P L; which was to be proved.

Otherwiſe, you may prove it by Numbers, thus: The Sum of 33 and 19 is 52, the half of which is 26, the Square of 26 is 676, and the Product of the Depth and Breadth is 627. The Difference of theſe two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the half of which is 7, whoſe Square is 49; which was to be proved.

Now, if this 49 be multiplied by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and thoſe Feet added to the true Content, the Sum

Sum will be equal to the Content found by the falſe Way mentioned.

See the Work of both.

33	Depth.	16.75	the Length.
19	Breadth.	49	the Sq. of $\frac{1}{2}$ Diff.
52	Sum.	15075	
		6700	
26	half.		
26		144)820.75(5.69	
156		1007	
52		1435	
676		139	
16.75			
3380			
4732			
4056			
676			
144)11323.00(78.63			
1243			
910			
460			
28			

	Feet.	
To	72.93	the true Content.
Add	5.69	the Part ſuperfluous.
Gives	78.62	equal to the Content by the falſe Way.

By Feet and Inches.

F.	I.			
0	7			
0	7			
0	4	1		
16	9			
5	5	4		
	3	0	9	
5	8	4	9	Part ſuperfluous.
72	11	2	3	True Content add.
78	7	7	0	equal to the Content by the falſe Way.

	F.	I.	
	2	2	
	2	2	
	4	4	
		4	4
	4	8	4
	16	9	
	75	1	4
	3	6	3
Falſe C.	78	7	7

To find how much in Length makes a Foot of any ſquared Timber.

Always divide 1728 (the ſolid Inches in a Foot) by the Area of the Baſe; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickneſs from End to End, whether it be ſquare, triangular, multangular, or round.

Example 1. If a Piece of Timber be 18 Inches ſquare; How much in Length will make a Foot ſolid?

```
  18
  18
 ---
 144
 18
 ---
324)1728(5 1/3 Inches; which is the Anſwer.
    1620
    ----
     108
```

By

By Scale and Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 18 to 323, the Square or Area of the Base; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches, and $\frac{1}{3}$ of an Inch.

Or thus: Extend the Compasses from 18 to 41.569, that Extent, turned twice over from 1, will at last fall upon $5\frac{1}{3}$, as before.

Note, That 41.569 is the square Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep, and 15 Inches broad; How much in Length will make a Foot?

```
   22
   15
  ---
  110
  22
  ---
  330)1728(5.23
         ----
          780
          1200
          ----
           210
```

Answer, 5 Inches and .23 Parts.

By Scale and Compasses.

Extend the Compasses from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There is a Line for this Purpoſe upon moſt ordinary Rules, with a little Table at the End of all ſuch Numbers as exceed the Length of the Rule, ſuch as this annexed.

0	0	0	0	9	0	11	3	9	Inches.
44	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of the Sq.

Here you ſee, if the Side of the Square be 1, the Length muſt be 144 Feet; if two Inches be the Side of the Square, it muſt be 36 Feet in Length, to make a ſolid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it againſt 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then if you take the Length of a Foot from the Line of Inches with your Compaſſes, and run the Compaſſes along the Piece from End to End, you will find how many Feet are contained in that Piece; or you may cut off any Number of ſolid Feet that ſhall be deſired; but if the Sides of the Pieces be unequal, find a mean proportional Number, as is before taught, by dividing the Diſtance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers into two equal Parts, and you will find the middle Point at 15; ſo is 15 Inches the geometrical mean Proportional ſought; then if you look for 15 upon the Line above-mentioned, you will find 7 Inches and a little above half to be the Length of a Foot.

§ III.

§ III. *Of unequal Squared Timber.*

BY unequal Squared Timber, I mean all ſuch as have unequal Baſes; that is, ſuch as is thicker at one End than at the other; and ſuch are moſt Timber-trees when they are hewn, and brought to their Squares.

The uſual Way to meaſure ſuch Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is ſomething near the Truth; but when there is a great Diſproportion between the Ends of the Piece, the Error is conſiderable. All ſuch Solids being the Fruſtums of Pyramids, the true Way of meaſuring them muſt be by Sect. VII. Chap. II. I ſhall give an Example or two, which I will work both by the true and falſe Ways, by which you will ſee the Difference.

Example 1. If a Piece of Timber be 25 Inches ſquare at the greater End, and 9 Inches ſquare at the leſſer End, and 20 Feet long; How many Feet of Timber are in that Tree?

```
      25
       9
     ———
Sum   34
     ———
Half  17  the Side of the Square in the Middle.
      17
    ————
     119
     17
    ————
     289
      20
    —————
144)5780(40.13
       200
        560
      ————
       128  Anſwer, 40.13 Feet, by the falſe Way.
```


By

By Rule II. Sect. VII. Chap. II.

```
 25        25
  9         9
———        ——
225        16 Difference of the Sides.
           16
           ——
           96
          16
          ———
        3)256 the Square.
        —————
          85.333
         225
        ————————
         310.333
              20
        —————————
   144)6206.660(43 101
        ———————————
          446
           146
            260
            ———
            116
```

Anſwer, 43.101 Feet, by the true Way; ſo that there is near 3 Feet Difference.

By Scale and Compaſſes.

Extend from 1 to 9, that Extent will reach from 25 (the ſame Way) to 225, the Rectangle of the Sides of the two Baſes; then the Difference between the ſaid Sides is 15: extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: Then extend from 144 to 310.333, that Extent

Extent will reach from 20 (the Length) to 43.1 Feet, the Content the true Way.

Extend the Compasses from 12 to 17 (the Side of the middle Square), that Extent will reach from 20 (the Length, being twice turned over) to 40.1 Feet, the Content by the false Way.

Example 2. If a Piece of Timber be 32 Inches broad, and 20 Inches deep, at the greater End, and 10 Inches broad and 6 deep, at the lesser End, and 18 Feet long; How many Feet of Timber are in that Piece?

Rule I. Sect. VII. Chap. II.

32	6
20	10
640	60
60	

38400 (195·959 mean Proportional.
1 — 640 the greater Base.
60 the lesser Base.

29)284
385)2300 — 895·959 the Sum.
3909)37500 — $6 = \frac{1}{3}$ the Height.
39185)231900
391909)3597500 — 5375·754

144)5375·754(37·33

1055
477
455

23

Add

```
Add {32         20} add.
    {10          6}
     --         --
Sum  42         26 Sum.
     --         --
Half 21         13 half.
     13
     --
     63
    21
   ---
   273 Area in the Middle.
    18 Length.
   ----
  2184
  273
  ----
144)4914(34.12
    ----
    594
     180
      360
    ----
      72
```

			Feet.
Anfwer	Content the true Way	———	37.33
	Content the falfe Way	———	34.12

By Scale and Compaffes.

Extend the Compaffes from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Bafe.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the leffer Bafe: Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the fquare Root of it, by dividing the Space between 1 and 38400 into two equal Parts, fo you will find the middle Point at 195.959, the Root fought; which is

is a mean Proportional between the greater and leſſer Areas; then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiplied by 6 (a third Part of the Length) by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the falſe Way, half the Sum of the Breadths is 21, which is the Breadth in the Middle; and half the Sum of the Depths is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Baſe: Then extend from 144 to 273, that Extent will reach from 18, the Length, to 34.12, the Content the falſe Way.

§ IV. *Of Round Timber, with equal Baſes.*

THE uſual Way to meaſure round Timber-trees, is to girt them about the Middle with a String, and take the fourth Part of that Girth for the Side of a Square, by which they meaſure the Piece of Timber as if it was ſquare.

But that this is an Error, I ſhall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which ſquared makes .0625; this they take for a mean Area, inſtead of .07958: Therefore the true Content always bears ſuch Proportion to the Content found by the aforeſaid cuſtomary falſe Way, as .07958 to .0625; which is nearly as 23 to 18; ſo that in meaſuring by that cuſtomary falſe Way, there is above one fifth Part loſt of what the true Content ought to be.

This

This Error, though it has been ſo often confuted, yet it is grown ſo cuſtomary in all Places, that there is little Hope of my prevailing with Men that are ſo wedded to it, to embrace the Truth: I ſhall therefore, in the following Examples, ſhew how to work both the true Way, and alſo the falſe or cuſtomary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference or Girth, and 18 Feet long; How many Feet of Timber are contained in it?

A fourth Part of 96 is 24

```
      24
      24
      --
      96
     48
     ---
     576  Area Baſe.
      18
     ---
    4608
    576
   -----
144)10368(72
    1008
    ----
      288
      288
      ---
      ...
```

Or thus,

```
 F.  I.
  2   0
  2   0
 ------
  4   0
 18   0
 ------
 72   0
```

Content the falſe Way, 72 Feet.

Then

Then the true Way.

```
            96
            96
          ----
           576
          864
          ----
          9216
        .07958
        ------
         73728
        46080
       82944
      64512
      ---------
      733.40928  the Area by Prob. 5. § IX. Ch. 1.
             18
     ----------
     586727424
     73340928
     ----------
144)13201.36704(91.67 Feet, the true Content.
    -----------
    241
     973
     1096
     ----
       88
```

By Scale and Compasses.

Extend from 12 to 24 (the fourth Part of the Girth) that Extent, turned twice over from 18 Feet (the Length) will at laſt fall upon 72 Feet, the Content the cuſtomary Way.

Extend from 42.54 to 96 (the Girth) that Extent will reach from 18 Feet, turned twice over, to 91.67 Feet, the true Content.

Example 2. If a Piece of Timber be 86 Inches Girth and 20 Feet long; How many Feet are contained therein?

The fourth Part of 86 is 21.5

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
			20
64	2	5	0

```
         21.5
         21.5
         ----
         1075
         215
        430
        ------
       462.25
           20
       ------
144)9245.00(64.2
    --------
      605
       290
      ----
        20
```

The Content the falſe Way, 64.2 Feet.

By

By the true Way.

```
        86
        86
      ----
       516
      688
      ----
      7396
    .07958
    ------
     59168
    36980
   66564
  51772
  ----------
  588.57368
         20
  ----------
144)11771.47360(81.74
  ----------
  251
  1074
    667
    ---
     91
```

The true Content, 81.74 Feet.

By Scale and Compasses.

Extend from 12 to 21.5, that Extent, turned twice over from 20, will reach at laſt to 64.2 Feet, the Content the falſe Way.

Extend from 42.54 to 86, that Extent, turned twice over from 20, will at laſt fall upon 81.74 Feet, the true Content.

The Cylindrical Proportions may be very eaſily wrought upon the Line of Numbers.

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppoſe the Diameter 22.6 Inches.

As 22.6 is to 46.9, ſo is 1 to a fourth Number, and that to the Length of a Foot in Inches 4.3.

Extend the Compaſſes from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

Note, That 46.9 is the Diameter of a Circle, of which the Area is 1728.

Problem 2. Having the Diameter in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Diameter 1.88 Feet.

Then, as 1.88 is to 1.128, ſo is 1 to a fourth Number: And ſo is that to the Length of a Foot in Foot-meaſure, .358.

Extend the Compaſſes from 1.88 to 1.128, that Extent, turned twice from 1, will reach to .358 Parts of a Foot.

Note, That 1.128 is the Diameter when the Side of the Square is equal to 1.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppoſe the Circumference 71 Inches.

Then, as 71 is to 147.36, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Inches, 4.3.

Extend the Compaſſes from 71 to 147.36, that Extent turned twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Note, That 147.36 is the Circumference of a Circle when the Area is 1728.

Problem 4. Having the Circumference in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe

Suppose the Circumference 5.92 Feet.

Then, as 5.92 is to 3.545, so is 1 to a fourth Number; and so is that to the Length of a Foot in Foot-measure, .358.

Extend the Compasses from 5.92 to 3.545, that Extent, turned twice over from 1, will fall upon .358 Parts of a Foot.

Note, That 3.545 is the Circumference when the Side of the Square is equal to 1.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches, or 13 Feet.

Then, as 1.128 is to 22.6, so is 156 to a fourth Number; and so is that to the Content in Inches, 62674.

Extend the Compasses from 1.128 to 22.6, that Extent, turned twice from 156, will fall upon 62674 Inches, the Content.

Problem 6. Having the Diameter in Foot-measure, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 is to 1.88, so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 1.128 to 1.88, that Extent, turned twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 is to 22.6, so is 156 to a fourth Number; and so is that to the Content in Feet, 36.27

Extend from 46.9 to 22.6, that Extent, turned twice from 156, will fall upon 36.27 Feet, the Content.

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 is to 22.6, so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 13.54 to 22.6, that Extent, turned twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle, when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppose the Circumference 71, and the Length 156 Inches.

Then, as 3.545 is to 71, so is 156 to a fourth Number; and so is that to 62674, the Content in Inches.

Extend the Compasses from 3.545 to 71, that Extent, turned twice from 156, will fall upon 62674, the Content.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 is to 5.92, so is 13 to a fourth Number; and so is that to 36.27.

Extend from 3.545 to 5.92, that Extent, turned twice from 13, will fall upon 36.27 Feet, the Content.

Problem

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 is to 71, ſo is 156 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 147.36 to 71, that Extent, turned twice from 156, will fall upon 36.27 Feet the Content.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 is to 71, ſo is 13 to a fourth Number; and ſo is that to the Content in Feet, 36.27.

Extend the Compaſſes from 42.54 to 71, that Extent, turned twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle, when the Area is 144.

§ V. *Of Round Timber, when the Baſes are unequal.*

THE uſual Way to meaſure Round Timber (as I ſaid before) is to take a fourth Part of the Girth in the Middle of the Piece, for the Side of a mean Square. But this Way I have proved to be erroneous in Timber that is all the Way of an equal Thickneſs; and it muſt be much more ſo in Timber that is tapering; and the more tapering it is, the greater is the Error: For to the Error in the laſt Section, there is added the Error in the third Section; therefore, to meaſure all ſuch Timber according to Art and Truth, ſuch a Piece ought to be conſidered as a Fruſtum of a Cone, and ſhould be meaſured by the Rules given in Sect. VIII. Chap. II. by which Rules the following Examples are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the leſſer End, and 36 Inches at the other End, and 24 Feet long; How many Feet of Timber are there in it?

	36	36	Subtract.
	9	9	
Rect.	324	27	Difference.
		27	
		189	
		54	
		3)729	the Square.
		243	One-third.
		324	Rectangle add
		567	

.7854

```
               .7854
                 567
               -----
               54978
              47124
             39270
            --------
A mean Area 445.3218
                  24
            --------
            17812872
            8906436
            --------
     144)10687.7232(74.22
            --------
            607
             317
              292
             ---
               4
```

Answer, 74.22 Feet.

Or thus, by Feet and Inches.

F.	I.		F.	I.		
3	0		2	3		Difference.
0	9		2	3		
2	3	Rect.	4	6		
			0	6	9	
			5	0	9	the Square.
			1	8	3	One-third.
			2	3	0	Rectangle added.
			3	11	3	a mean Square.

Then,

Then, As 14 is to 11, ſo is 3 F. 11 I. 3 P. to the Area.

	F.	I.	P.	
	3	11	3	
			11	
7)	43	9	9	
2)	6	2	3	
	3	1	1	6
				6
	18	6	9	0
				4
	74	3	0	0

Here, inſtead of dividing by 14, I divide by 7 and by 2, becauſe twice 7 is 14.

And inſtead of multiplying by 24 Feet, the Length, I multiply by 6 and 4, becauſe 6 times 4 is 24.

By Scale and Compaſſes this is too troubleſome.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, 32 Inches Circumference at the other End, and 21 Feet long; How many Feet of Timber are contained in that Piece?

```
 136          136
  32           32
----         ----
 272          104  Difference.
408           104
----         ----
4352          416
             1040
            -----
          3)10816  the Square.
          --------
          3605.333  One-third.
          4352      Rectangle add.
          --------
          7957.333  Square of a mean Cir.
            .07958
          --------
          63658664
         39786665
        71615997
       55701331
      ------------
      633.24456014  the mean Area.
                21
      ------------
        63324456014
      126648912028
      -------------
     13298.13576294
      144)13298.13(92.34
          ---------
           338
            501
             693
            ----
             117
```

Answer, 92.34 Feet.

By

By Feet and Inches, thus:

F.	I.	
11	4	
2	8	
22	8	
7	6	8
30	2	8

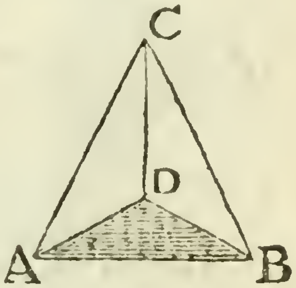
	F.	I.			
	8	8			Difference.
	8	8			
	69	4			
	5	9	4		
3)	75	1	4		the Square.
	25	0	5	4	
	30	2	8	0	
	55	3	1	4	the Sq. of the Circum.
	F.	I.	P.	S.	
88 : 7 ::	55	3	1	4	: the mean Area.
				7	
11)	386	9	9	4	
8)	35	1	11	9	
	4	4	8	11	the mean Area.
				7	
	30	9	2	5	
				3	
Facit	92	3	7	3	

§ VI. *Of the Five Regular Bodies.*

THESE Bodies may all be meaſured by the fourth Section of Chap. II. except it be the Cube, or Haxaedron, which is already meaſured in Section I. of that Chapter.

1. *Of the* TETRAEDRON.

A Tetraedron is a Solid, contained under four equal and equilateral Triangles.



Let A B C D be a Tetraedron, whoſe Side is 12 Inches, the perpendicular Height 9.798 Inches.

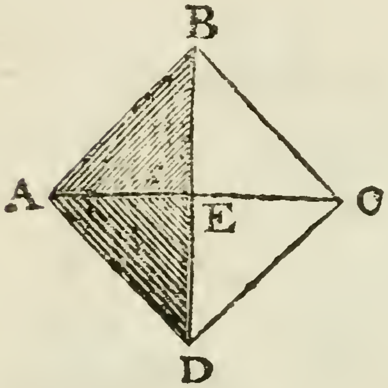
By Sect. V. Chap. I. the Area of the Triangle will be found 62.352; a third Part of it is 20.784, which multiplied by the perpendicular Height, the Product is 203.641632 ſolid Inches, the Content.

10.392	the Perpendicular of the Triangle.
6	Half the Side.
62.352	Area of the Triangle.
20.784	one-third Part.
9.798	the perpendicular Height.
166272	
187056	
145488	
187056	
203.641632	

The

The ſuperficial Content is four times the Area of the Triangle, *viz.* 249.408 Inches, becauſe there are four Triangles.

2. *Of the* OCTAEDRON.



The Octaedron is a Body contained under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whoſe Side is 12 Inches; the Content ſolid and ſuperficial is required.

An Octaedron is compoſed of two quadrangular Pyramids joined together by their Baſes; therefore, if the Area of the Baſe be multiplied into a third Part of the Length of both Pyramids, the Product will be the ſolid Content.

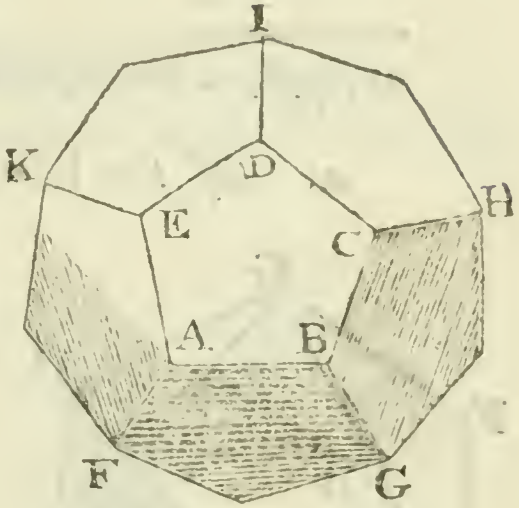
5.6568	a third Part of the Length.
144	Area of the Square Baſe.
226272	
226272	
56568	
814.5792	the Solidity.

The ſuperficial Content will be juſt double to that of the Tetraedron, *viz.* 498.816, becauſe the Side of this is ſuppoſed to be equal to the Side of that, and becauſe the Octaedron is contained under eight Triangles, and the Tetraedron but under four.

3. *Of the* DODECAEDRON.

The Dodecaedron is a ſolid Body, contained under twelve pentangular Planes.

Let ABCDEFG HIK be a Dodecaedron, each Side of it being 12 Inches; the Content, ſolid and ſuperficial, is required.



The Solidity of the Dodecaedron is composed of twelve pentangled Pyramids, whoſe Vertexes all meet in the Center. Therefore, if we find the Solidity of one of thoſe Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

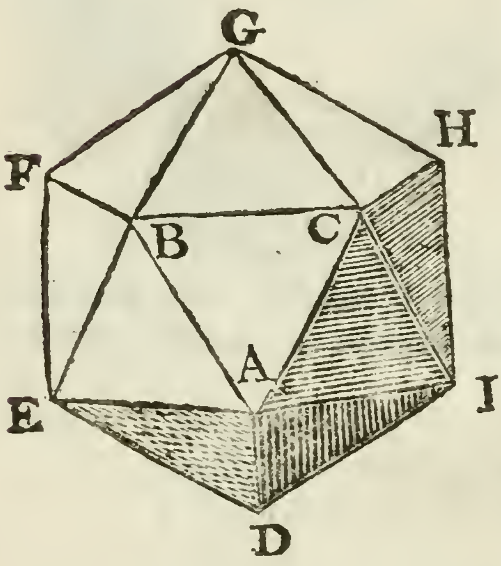
8.258292	
30	half Sum of the Sides.
247.748760	Area of the Pentagon.
60454.4	a third Part of 13.36219 inverted.
99099504	
9909950	
1238744	
99099	
1486	
1103.48783	Content of one Pyramid.
12	
13241.85396	the Solidity of the Dodecaedron.

 If

If the Area of the Pentagon be multiplied by 12, the Product will be the ſuperficial Content.

$$\begin{array}{r} 247.74876 \\ 12 \\ \hline 2972.98512 \end{array}$$ the ſuperficial Content.

4. *Of the* ICOSAEDRON.



The Icoſaedron is a ſolid Body, contained under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icoſaedron, each Side of which is 12 Inches; the Content, ſolid and ſuperficial, is required.

The Icoſaedron is compoſed of twenty triangular Pyramids, with their Vertexes all joining in the Center.

Therefore, if the ſolid Content of one Pyramid be multiplied by 20, the Product is the whole ſolid Content of the Icoſaedron.

10.39224 the Perpendicular of the Triangle.
6 Half the Side.

62.35344
20

1247.06880 Superficial Content.

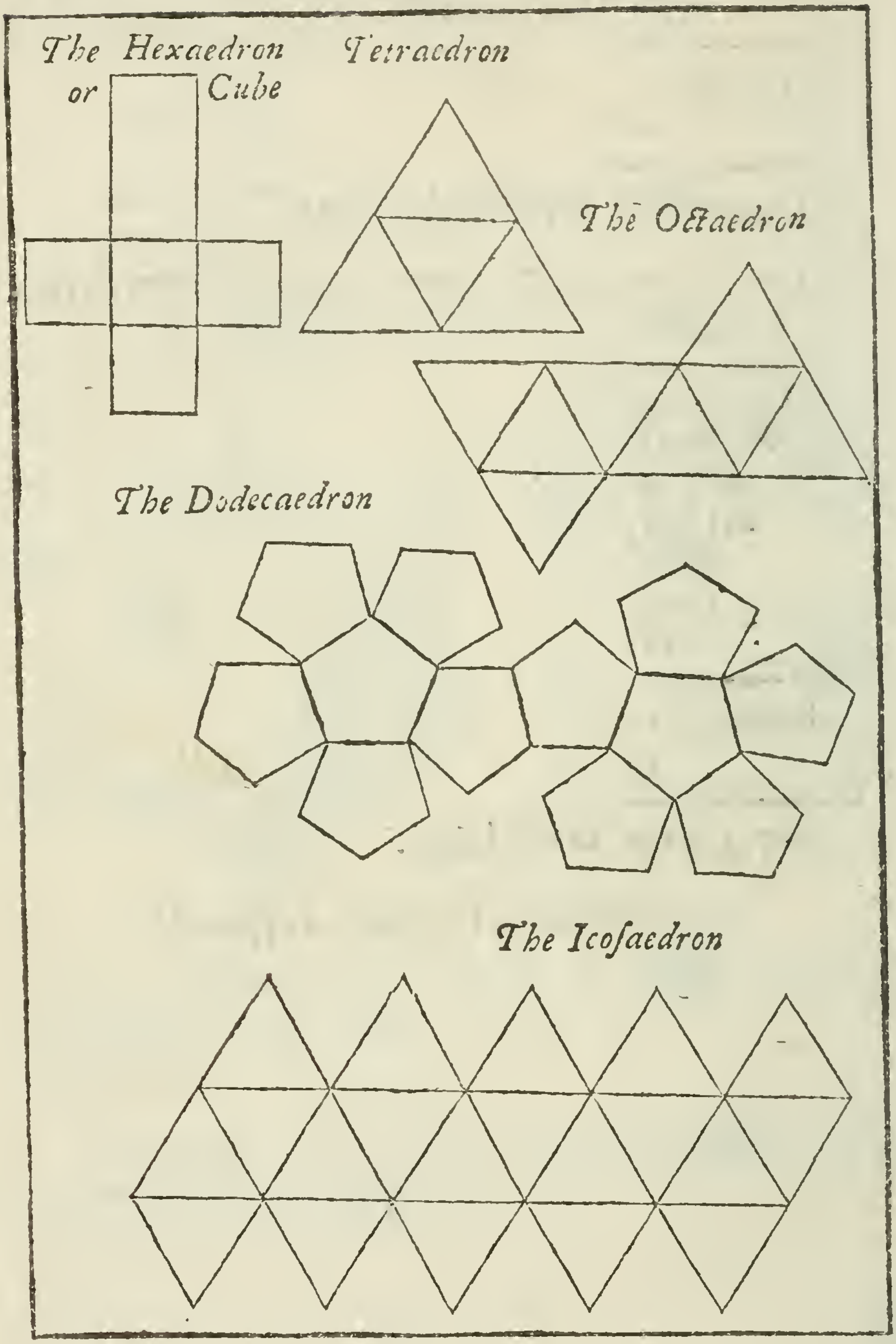
3.0230456 third Part of the Altitude of the Pyram.
44353.26

181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 the Solidity.

The superficial Content, 1247.0688.



By theſe Figures you may cut theſe Bodies in fine Paſteboard, cutting all the Lines half through, and ſo turn them up and glue them.

A TABLE *ſhewing the Solidity and ſuperficial Content of any of the Regular Bodies, the Side being* 1, *or Unity.*

The Names of the Bodies.	The Solidity.	Superficies.
Tetraedron	0.1178511	1.732051
Octaedron	0.4714045	3.464102
Hexaedron	1.0000000	6 000000
Icoſaedron	2.181695	8.660254
Dodecaedron	7.663119	20.645729

By this Table, the Content, either ſuperficial or ſolid, of any of theſe Bodies, may very readily be found; for all like ſuperficial Figures are in Proportion one to another, as are the Squares of their like Sides: Therefore it will be, As the Square of 1, (which is 1) is to the ſuperficial Content in the Table; ſo is the Square of the Side of the like Body, to the ſuperficial Content of the ſame Body. Therefore, if the Number in the Table be multiplied by the Square of the Side given, the Product is the ſuperficial Content required.

Again; all like Solids are in Proportion to each other, as the Cubes of their like Sides. Therefore it will be, As 1 (which is the Cube of 1) is to the ſolid Content in the Table, ſo is the Cube of the Side given, to the ſolid Content required. Therefore, if the Number in the Table be multiplied by the Cube of the given Side, the Product will be the ſolid Content of the ſame Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before); What is the Content folid and fuperficial?

```
     7.663119 the tabular Number.
         1728 the Cube of the Side.
     --------
     61304952
    15326238
   53641833
   7663119
  -----------
 13241.869632 the folid Content, nearly the fame
                                     (as before.
            20.645729 the tabular Number.
                  144 the Square of the Side.
            ---------
             82582916
            82582916
           20645729
           -----------
           2972.984976 the fuperficial Content.
```

By Scale and Compaffes.

Extend from 1 to 12 (the Side) that Extent being turned three times over from 7.63119, will at laft fall upon 13241.86, &c. the folid Content.

And if you apply the fame Extent twice from 20.645729, it will at laft fall upon 2972.98, &c. the fuperficial Content.

Example 2. If the Side of an Octaedron be 20 Inches; What is the Content solid and superficial?

.4714045	the tabular Number.
8000	the Cube of the Side.
3771.2360000	the solid Content.
3.464102	the tabular Number.
400	the Square of the Side.
1385.640800	the superficial Content.

By Scale and Compasses.

Extend from 1 to 20, that Extent, turned three times over from .4714045, will at last fall upon 3771.236, the solid Content. The same Extent, turned twice over from 3.464, &c. will at last fall upon 1385.64, the superficial Content.

§ VII. *How to measure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy or uneven, and you desire to find the Solidity, put the Solid into any regular Vessel, as a Tub, a Cistern, or the like, and pour in as much Water as will just cover it; then take out the Solid, and measure how much the Fall of the Water is, and so find the Solidity of that Part of the Vessel.

Example. Suppoſe a Piece of Wood or Stone to be meaſured, and ſuppoſe a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and covered with Water; then, when the Solid is taken out, ſuppoſe the Fall of the Water 14 Inches; Square 32, and multiply the Square by .7854, the Product will be 804 2496, the Area of the Baſe; which multiplied by 14, the Depth or Fall of the Water, and the Product is 11259.49, *&c.* which divided by 1728, the Quotient is 6.51 Feet; and ſo much is the ſolid Content required.

CHAP. V.

Practical Questions in MEASURING.

Question 1. IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contained in it?

F.	I.	
47	9	
18	6	
376	0	
47		
23	10	6
9	0	0
4	6	0
883	4	6

F.
47·75
18·5
23875
38200
4775
9)883·375
98.1

Answer, 98 Yards 1 Foot.

Quest.

Queſt. 2. There is a Room, whoſe Length is 21.5 Feet, and the Breadth 17.5 Feet, which is to be paved with Stones, each 18 Inches ſquare; I demand how many ſuch Stones will pave it?

21.5	1.5
17.5	1.5
1075	75
1505	15
215	
	2.25 Area of one Stone.
2.25)376.25(167	
1512	
1625	
50	Anſwer, 167 Stones.

Queſt. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad, to be hung with Tapeſtry that is Ell-broad; I deſire to know how many Yards will hang the ſaid Room?

From the Content of the Room, ſubtract the Content of the Windows, and divide the Remainder by the ſquare Feet in a Yard of Tapeſtry.

3·75

3.75	109.75	Length.	5.75
3	9.25	Breadth.	6.5
11.25	54875		2875
	31950		3450
	98775		37.375
	1015.1875	Content of the Room.	2
	74.75	Content of the Wind. sub.	74.750

11.25)940.4375(83.59
4043
6687
10625
500

Anſwer, 83.59 Yards.

Queſt. 4. If the Axis of a Globe be 27.5 Inches; I demand the Content ſolid and ſuperficial.

3.1416
27.5

157080
219912
62831

86.39400 the Circumference.

86.394

```
      86.394  the Circumference.
        27.5  the Diameter.
    --------
      431970
     604758
    172788
    --------
  6)2375.8350  the superficial Content.
    --------
     395.9725  a sixth Part.
        27.5
    --------
    19798625
   27718075
   7919450
    --------
  10889.24375
Answer, { 6.3   Feet solid.
        { 16 49 Feet superficial.
```

Quest. 5. There is the Frustum of a Globe, the Diameter of whose Base is 24 Inches, and its Altitude 10 Inches; What is the Content solid and superficial?

Find the Superficies as directed in Page 190, and the Solidity by the first Theorem in Page 192.

```
  24          .7854         .7154        10
  24            576           100        10
 ----        ------     ---------       ----
  96          47124       78.5400       100
 48          54978
 ----        39270
 576        --------
            452.3904 } add.
             78.54   }
            --------
            530.9304  the Curve Superficies.
            452.3904  the Base add.
            --------
        98)3.3208 { the whole superficial Con-
                  { tent in Inches.
                3                       12
```

12×12=144

3

———

432

100 the Square of the Alt. add.

———

532 the Sum.

10 multiply by the Alt.

———

5320

5236 multiply.

———

31920

15960

10640

26600

———

2785.5520 the Solidity in Inches.

Quest. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; How many Tons of Timber are contained in that Tree?

	F.	I.			
4)	18	6			the Girth.
	4	7	6		a 4th Part.
	4	7	6		
	18	6	0		
	2	8	4	6	
		2	3	9	
	21	4	8	3	Here I multiply by 6 and by
				6	4, because 6 times 4 is 24.
	128	4	1	6	
				4	
4|0)	51|3	4	6	0	
	12	33			

Answer, 12 Tons 33 Feet 4 Inches 6 Parts.

Note, That 40 Feet of Timber is a Ton, and 50 Feet a Load.

Note also, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Chord of Fire-wood, that is, 128 cubical Feet.

Quest. 7. There is a Cellar to be dug by the Floor, the Length of which is 33 Feet 7 Inches, and the Breadth 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar?

F.	I.		
33	7		the Length.
18	9		the Breadth.
264			
33			
16	9	6	
8	4	9	
9	0	0	
1	6	0	
629	8	3	
5	9	0	the Depth.
3148	5	3	
314	10	1	
157	5	1	
324)3620	8	5(11	
380			
56			

Answer, 11 Floors, 56 Feet, 8 Inches, 5 Parts.

Note, That 18 Feet square and a Foot deep is a Floor of Earth, that is, 324 solid Feet.

Quest.

Quest. 8 There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; How many Squares of Tiling are contained in it?

F.	I.	
48	9	
35	6	
240		
144		
24	4	6
17	6	0
8	9	0
17|30	7	6

Answer, 17 Squares, 30 Feet, $7\frac{1}{2}$ Inches.

Quest. 6. There is a Cone, the Diameter at the Base being 42 Inches, and the perpendicular Height 94 Inches; and it is required to cut off two solid Feet from the Top-end of it; I demand what Length upon the Perpendicular must be cut off?

```
      42     1728      94
      42        2      94
     ---     ----     ---
      84     3456     376
     168              846
    ----             ----
    1764             8836 Square.
    .7854              94
    ----            -----
    7056            35344
   8820            79524
  14112            ------
 12348             830584 the Cube.
---------
1385.4456
       94
---------
 55417824
124690104
-----------
3)130231.8864
-------------
  43410.6288
```

All the solid Bodies are in the triplicate Ratio of their homologous Sides, by *Eucl.* 12, 12; 12, 18; and 11, 33: Therefore it will be,

```
Solidity of the Cone.   Cub. Alt.   Solidity of 2 Feet.
43410.6288      :     830584    : :   3456   :
                        3456
                     -------
                     4983504
                    4152920
                   3322336
                  2491752
                  ----------
43410.6288)2870498304(66124 the Cube of the
    . . .  ----------                (Length.
           265860576
            539680320
            1055740320
             1875277440                       66124
```

```
66124(40.43
64
─────────
 2124000 Resolvend.
─────────
 12
48
────
492 Divisor.
─────
  120
4800
─────
48120 Divisor.
──────
     64
  1920
19200
──────
1939264 Subtrahend.
─────────
184736000 Resolvend.
─────────
     1212
  489648
──────
  4897692 Divisor.
─────────
        27
    10908
 1468944
─────────
147003507 Subtrahend.
─────────
 37732493
```

Answer, The Length upon the Perpendicular must be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet were to be cut off from the Bottom, or greatest End, then from 43410.6288 subtract 3456, and the Remainder is 39954.6288. Then say,

```
43410.6288 : 830584 : : 39954.6288
                            830584
                        ----------
                        1598185152
                       3196370304
                      1997731440
                    11986388640
                   3196370304
                ------------------
     43410.6288)33185675407.2192(764459(91.4
       .....  --------------    729
                279823524      -----
                 19359751      35459
                  1995500     ------
                   259075         27
                    42022        243
                   ------       -----
                     2952       2457
                               -----
                                 271
                               243
                               ------
                               24571
                               --------
                               10888000
                               --------
                                    273
                                 24843
                               --------
                                 248703
```

Answer, It must be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Quest.

Quest. 10. If a ſquare Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Baſe be 21 Inches, and the Side of the Square of the leſſer Baſe be 3 Inches; How far muſt I meaſure from the greater End, to cut off five ſolid Feet?

Firſt find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18: Then,

Diff. Length. great. Length.
As 18 : 12 : : 21 : 14.

So I find the whole Length of the Pyramid to be 14 Feet, or 168 Inches.

The ſolid Content of the whole Pyramid is 24696 Inches, and the ſolid Content of 5 Feet is 8640; which ſubtracted from 24696, there remain 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

24696 : 4741632 : : 16056 :
3082752, whoſe Cube Root is 145.54; ſubtract this Root from 168 (the Length) and there remains 22.46 Inches, which is the Length of 5 ſolid Feet at the great End.

Quest. 11. Three Men bought a Grind-ſtone of 40 Inches Diameter, which coſt 20 Shillings; of which Sum the firſt Man paid 9 Shillings, the ſecond 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man muſt grind down, proportionable to the Money he paid?

All Circles are in the duplicate Ratio of their Diameters, by *Eucl.* 12, 2.

Square the Semidiameter, which makes 400. Then

s. Square. s.
20 : 400 : : 9 : 180.

This 180 is the Square of the Semidiameter of the Circle belonging to the firſt Man.

s. s.
And, 20 : 400 : : 6 : 120.

This 120 is the Square of the Semidiameter of the Circle belonging to the ſecond.

s. s.
And, 20 : 400 : : 5 : 100.

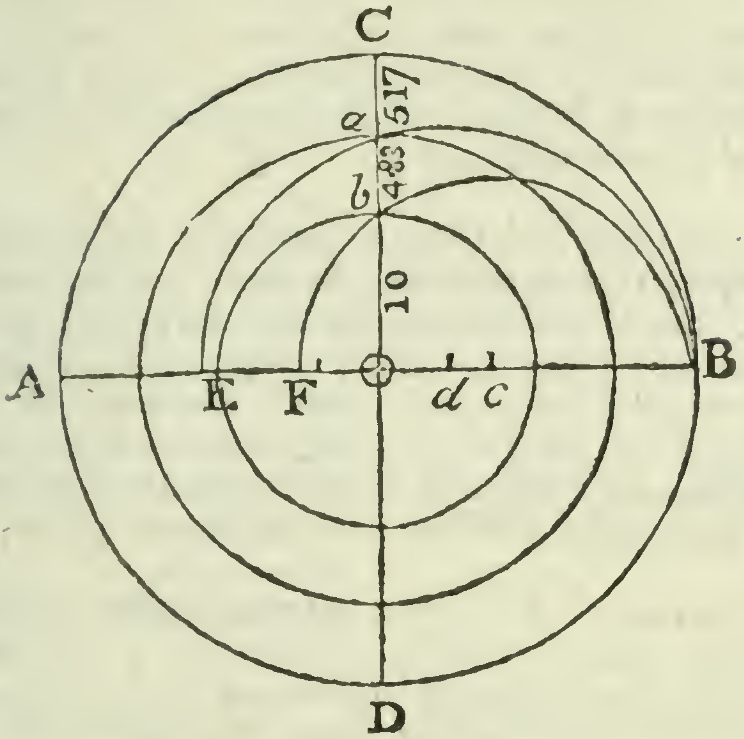
This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then, from 400 (the Square of the Semidiameter of the Stone) ſubtract 180; and there remains 220, whoſe ſquare Root is 14.83 Inches; which ſubtracted from 20 Inches, (the Semidiameter,) there remain 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which muſt be ground down by the firſt.

Then, from 220 ſubtract 120, and there remains 100, whoſe ſquare Root is 10; ſubtract that from 1; 83, and there remains 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the ſecond Man. The third muſt grind down the Remainder, which is 10 Inches, the ſquare Root of 100.

This Queſtion may very eaſily and ſpeedily be performed geometrically, as in the annexed Scheme.

First,



First, upon the Center ⊙ ſtrike the Circle ABCD, and croſs it at right Angles with the two Diameters AB and CD: Then divide the Semidiameter A ⊙ (which ſuppoſe 20) in Proportion to 9 s. 6 s. and 5 s. (the ſeveral Sums paid by the three Men) by the Point E and F; ſo ſhall AE be 9, EF 6, and F ⊙ 5: Then divide EB into two equal Parts in d, and upon d, as a Center, ſtrike the Semicircle E a B; and divide FB into two equal Parts in c, and upon c, as a Center, with the Radius cF, ſtrike the Semicircle FbB: So have you the Semidiameter ⊙ C divided into three ſuch Parts as the Stone ought to be divided; and Circles, ſtruck through thoſe Points, will ſhew how much each Man muſt grind for his Share.

Queſt. 12. A Gard'ner he had an upright Cone,
Out of which ſhould be cut him a Rolling-ſtone,
The biggeſt that e'er it could make:
The Maſon he ſaid, That there was a Rule
For ſuch ſort of Work, but he had a thick Skull:
Now help him for Pity's Sake.

Anſwer, It muſt be cut at one-third Part of the Altitude.

Queſt.

Queſt. 13. There is a Ciſtern, whoſe Depth is ſeven Tenths of the Width, and the Length is 6 times the Depth, and the ſolid Capacity is 367.5 Feet; I demand the Depth, Width, and Length, and how many Buſhels of Corn it will hold?

Firſt, you muſt find three Numbers in Proportion to the Depth, Width, and Length, thus: Suppoſe the Depth 7, then the Width will be 10, and the Length 42; which multiplied together, the Product is 2940, which is the ſolid Inches in a Ciſtern, whoſe Depth is 7, Width 10, and Length 42. But the ſolid Inches in the Queſtion are 635040 ($=367.5\times1728$) then the Cube of the ſuppoſed Width is 1000. So it will be,

2940 : 1000 : : 635040 : 216000, whoſe Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiplied together, the Product will be 635040. If theſe ſolid Inches be divided by 2150.42, and the Quotient is $295\frac{66315}{215042}$ Buſhels, or 36 Quarters 7 Buſhels 1 Peck 4 Pints. And ſo much will the Ciſtern hold.

Queſt. 14. Suppoſe, Sir, a Buſhel be exactly round,
Whoſe Depth being meaſur'd, 8 Inches is found;
If the Breadth 18 Inches and half you diſcover,
This Buſhel is legal all *England* over.
But a Workman would make one of another Frame;
Sev'n Inch and a half muſt be the Depth of the ſame;
Now, Sir, of what Length muſt the Diameter be,
That it may with the former in Meaſure agree?

```
      18.5
      18.5
      ----
       925
     1480
     185
    ------
    342 25 the Square.
     .7854
    ------
    136900
   171125
  273800
 239575
 ----------
 268.803150
          8
 ----------
2150.425200 the Solid Inches in a Bushel.

      7.5)2150 4252(286.72336
              ...
           ----------
             650
               504
                 542
                   175
                     252
                       270
                       450
                      ----
                       ...
```

.7854)286.72336

51103
39793
5236

)365.0666(19.107
1

29)265
261

381)406
381

38207)256666

Answer, The Diameter must be 19.107 Inches, if the Depth be 7.5 Inches.

Quest. 15. In the Midst of a Meadow well stored with Grass,
I took just an Acre to tether my Ass;
How long must the Cord be, that feeding all round,
He may'nt graze less nor more than his Acre of Ground?

By Problem 10. Section IX. Chap. I. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre.
66 Feet, the Breadth of an Acre.

3960
3960

43560 the square Feet in an Acre.

```
As 1 : 1.2732 : : 43560
               43560
              ------
              763920
             63660
            38196
           50928
           ---------
           . . .
           55460.5920(235.5 Diameter.
           4          ------
           ---        117.75 half.
        43)154
           129
          -----
       465)2560
           2325
          -----
     4705)23559
          23525
          -----
             34
```

Answer, The Cord must be 117 Feet and 9 Inches.

Quest. 16. A Maltster has a Kiln, that is 16 Feet 6 Inches square; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much square the new one must be?

```
       16.5
       16.5
       ----
        825
       990
      165
     ------
     272.25 the Area of the old one.
          3
     ------
     .  .  .
     816.75(28.57
     4
     ----
  48)416
     384
     -----
 565)3275
     2825
     ------
5707)45000
      39949
      -----
       5051
```

Answer, The Side of the new one must be 28 Feet and near 7 Inches.

Quest. 17. If a round Cistern be 26.3 Inches diameter, and 52.5 Inches deep; How many Inches diameter must a Cistern be to hold twice the Quantity, the Depth being the same? And how many Ale Gallons will each Cistern hold?

```
        26.3
        26.3
        ----
         789
       1578
       526
      ------
      691.69 Square.
           2
      ------
      .  .
     1383.38(37.19
      9
   -----
   67)483
      469
     -----
  741)1438
       741
     ------
 7429)69700
      66861
     ------
       2839
```

The Diameter of the greater is 37.19 Inches.

```
        691.69 the Square of the leſſer Ciſ-
         .7854               (tern's Diameters.
        ------
        276676
       345845
      553352
     484183
     ----------
     543.253326 Area o      Baſe.
           52.5
     ----------
     2716266630
    1086506652
   2716266630
   ------------
   28520.7996150 ſolid Content in Inches.
   ------------
282)28520.799(101.137 Gallons.
   ------------
     320
      387
      1059
       2139
       ----
        165
```

Note, That 282 ſolid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

You

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

```
359.05)1383.38(3.853        3.853
       -------               52.5
       306230               -----
        18990               19265
         1037               7706
                           19265
                           -----
```

The Content of the greater, 202.2825 Gallons.

Quest. 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches; How many Ale Gallons are contained therein?

```
 25              32
 25              32
----             --
125              64
50               96
----            ----
625             1024 Square of the Bung Diameter.
                1024 the same.
                 625 Square of the Head Diameter.
                ----
       1077)2673(2.48          2.48
           -----                 40
            5190              -----
             8820             99.20
             ----
              204
```

Answer, 99.2 Gallons.

Otherwise you may find a mean Diameter, and work by Scale and Compasses, thus: Subtract 25 from 32, and there remains 7, which multiplied by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compasses from 18.95 to 29.9, that Extent turned twice from 40 (the Length) will fall upon 99.6 Gallons; something more than before.

Quest. 19. There is a Stone, 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickness of another of the same Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then (by *Eucl.* 11. 33)

217 : 8000 : : 1000 : 36866.3594, whose Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then say,

20 : 33.28 : : 15 : 24.96
20 : 33.28 : : 8 : 13.312

Answer, The Length 33.28, The Breadth 24.96, The Thickness 13.312 Inches.

Quest. 20. If an Iron Bullet, whose Diameter is 4 Inches, weighs 9 Pounds; What will be the Weight of another Bullet (of the same Metal) whose Diameter is 9 Inches?

The Cube of 4 is 94, and the Cube of 9 is 729. Then (by *Eucl.* 12. 18.)

64 *lb.* : 9 : : 729 : 102.515 *lb.*

Answer, It weighs 102 *lb.* 8 *oz.* 4 *dr.* *ferè.*

Quest.

Quest. 21. There is a square Pyramid of Marble each Side of its Base is 5 Inches, and the Height 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like square Pyramid, each Side of whose Base is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000. Then (by *Eucl.* 12. 12.)

lb. *lb.*
125 : 12.25 : : 27000 : 2646.
Answer, The Weight is 2646 Pounds.

Quest. 22. There is a Ball or Globe of Marble, whose Diameter is 6 Inches, and its Weight 11 Pounds; What will be the Diameter of another Globe of the same Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,
lb. *lb.*
11 : 216 : : 500 : 9818.1818, &c.
Whose Cube Root is 21.4 Inches, the Diameter sought.

Quest. 23. There is a Frustum of a Pyramid, whose Bases are regular Octagons; each Side of the greater Base is 21 Inches, and each Side of the lesser Base is 9 Inches, and its Length is 15 Feet; I demand how many solid Feet are contained in it?

4.8284

```
    4.8284  the tabular Number, Page 89.
       237  the Square of a mean Side.
    ------
    337988          21          12
   144852            9          12
   96568           ---         ---
  ---------        189      3)144
 1144.3308          48       ---
        15         ---        48
  ---------        237
  57216540
 11443308
 ----------
144)17164.9620(119.2
     ....
    -----------
    276
    1324
     289
     ---
       1
```

Answer, 119.2 solid Feet.

Quest. 24. There is a Frustum of a Cone, the Diameter of the greater Base is 36 Inches, and the Diameter of the lesser Base is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and solid Content of the whole Cone, and also the solid Content of the given Frustum?

First, Find the Length of the whole Cone, thus:

```
From   36
Subtr. 20
       --
       16 : 215 : : 36 : 483.75.
```

So the Length of the whole Cone is $483\frac{3}{4}$ Inches, and $\frac{1}{3}$ of it is 161.25 Inches.

Then

Then find the Content of the whole Cone.

36	1017.8784
36	52.161
216	10178784
108	6107270
	101788
1296	20357
.7854	5089
5184	1728)164132.88(94.98 Feet.
6480	
10368	8612
9072	17008
	14568
1017.8784 Area Base.	
	744

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the solid Content of the Top-part that is wanting: Thus,

Whole Length of the Cone ——	483.75
Length of the Frustum ——	215.
Length of the Top-part ——	268.75

.7854

```
          .7854   the Area of Unity.
            400   the Square of 20.
       ----------
      3)314.1600  Area of the lesser Base.
       ----------
         104.72   a third Part.
         268.75   Altitude of the Top-part.
       ----------
          52360
         73304
        83776
       62832
      20944
    ------------
1728)28143.5000(16.28 Feet.
    -----
     10863
       4955
       14990
     ------
       1166
```

	Feet.
Content of the Whole	94.98
Content of the Top-piece	16.28
Content of the Frustum	78.7

Quest. 25. If the Top-part of a Cone contains 26171 solid Inches, and 200 Inches in Length, and the lower Frustum thereof contains 159610 solid Inches; I demand the Length of the whole Cone, and the Diameter of each Base?

```
    200          159610 } add.
    200           26171 }
  -------        --------
  40000          185781  the Sum.
      200
  -------
  8000000
```

26171

26171 : 8000000 : : 185781 : 56789881.93, whose Cube Root is 384.3 Inches, the Length of the whole Cone.

Then find the Diameter of the leſſer Baſe, thus:

```
200)26171
    ------
    130.855
          3
    -------
    392.565  Area of the leſſer Baſe.
```

Then, by Prob. X. Sect. IX. Chap. I.

```
1 : 1.2732 : : 392.565
                1.2732
               -------
                785130
               1177695
              2747955
              785130
             392565
             ----------
             .  .  .  .  .
             499.8137580(22.356
             4
             --
           42)99
              84
              ---
          443)1581
              1329
              ----
         4465)25237
              22325
              -----
        44706)291258
              268236
              ------
               23022
```

Again;

Leſſer Len. Leſſ. Diam. Gr. Leng. Gr. Diam.
Again; 200 : 22.356 : : 384 : 42.957.

		Inches.
Anſwer,	The Length of the whole Cone	384.3
	The Diameter of the greater Baſe	42.957
	The Diameter of the leſſer Baſe	22.35

Queſt. 26. There is a Fruſtum of a Cone, whoſe ſolid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears ſuch Proportion to the leſſer as 5 to 2; I demand the Diameters?

$5 \times 5 = 25$
$2 \times 2 = 4$
$5 \times 2 = 10$

The Sum 39

3)12

4)20(5 Feet.

Theſe 5 Feet are the Triple of a mean Area.

Then, 1 : 1.27324 : : 5 : 6.3662.

So the triple Square of a mean Diameter is 6.3662.

Then, 39 : 6.3662 : : 25 : 4.080897.

This 4.080897 is the Square of the greater Diameter, whoſe ſquare Root is 2.020123 Feet; which is 24.24147 Inches. Then,

5 : 24.24147 : : 2 : 9.69659

So the greater Diameter is 24.24147, and the leſſer Diameter is 9.69659 Inches.

Queſt. 27. There is a Room of Wainſcot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high (being girt over the Mouldings;) there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-boards and Top and Bottom-boards of each Window,

Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-case 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainscot are contained in that Room?

F.	I.		
129	6		
16	9		
782	0		
129			
64	9		
32	4	6	
2169	1	6	

F.	I.		
7	3		
4	6		
29	0		
3	7	6	
32	7	6	
16	3	9	half.
48	11	3	
		2	
97	10	6	

F.	I.		
3	3		
7	0		
22	9		
11	4	6	half.
34	1	6	

F.	I.	
24	6	
1	9	
24	6	
18	4	6
42	10	6
		2
85	9	0

F.	I.	
7	3	
5	6	
36	3	
3	7	6
39	10	6
		2
79	9	0

F.	I.	
3	6	
7	0	
24	6	add.
79	9	
104	3	deduct.

The

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-boards, &c.	85	9	0
The Sum	2386	10	6
The Window-lights and Door-case deduct	104	3	0
	9)2282	7	6
	253	5	

Answer, 253 Yards 5 Feet.

Quest. 28. There is a Wall which contains 18225 Cube Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; What is the Length, Breadth, and Height?

Suppose the Breadth 2, then the Height must be 10, and the Length 80; which three Numbers multiplied together, the Product will be 1600, and the Cube of 2 is 8: Then say,

1600 : 8 : : 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5, the Height; and 8 times 22.5 is 180, the Length.

Quest. 29. There is a May-pole, whose Top-end was broken off by a Blast of Wind, and the Top-end, in falling, struck the Ground at 15 Feet Distance from the Top of the May-pole, the broken Piece was 39 Feet; Now I demand the Length of the May-pole?

By *Eucl.* 1. 47, the Square of the Hypotheneuse of a right-angled Triangle is equal to the Sum of the Squares of the Base and Perpendicular.

There-

Therefore, from the Square of 39 subtract the Square of 15, the Square Root of the Remainder is the Piece standing; to which, add the Piece broken off, and you have the whole Length.

39	15
39	15
351	75
117	15
1521	225
225	

1296(36
9
66)396
396

The Piece standing is 36 Feet.
The Piece broken off is 39 Feet.

The whole Length 75.

Question 30.

A May-pole there was, whose Height I would know,
The Sun shining clear, strait to work I did go:
The Length of the Shadow, upon level Ground,
Just sixty-five Feet, when measur'd, I found;
A Staff I had there, just five Feet in Length;
The Length of its Shadow was four Feet One-tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're desir'd to show.

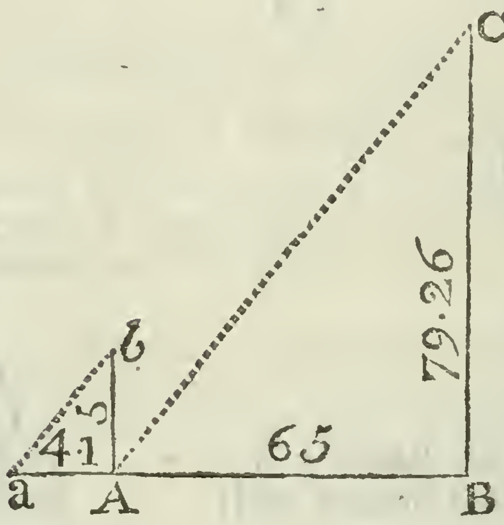
By *Eucl.* 6. 4.

a A : A b : : AB : B C

That is,

4.1 : 5 : : 65 : 79.26.

So I find the Height of the May-pole to be 79 Feet, and a little above 3 Inches.



Here A B represents the Length of the Shadow of the May-pole, and B C the May-pole; a A the Shadow of the Staff, and A b the Staff.

Quest. 31. What will be the Diameter of a Globe, when the Solidity and superficial Content of it are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the superficial Content, we must multiply 3.1416 by the Square of the Axis or Diameter, and the Product is the superficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the

the Product is the ſolid Content; therefore, becauſe .5236 is a ſixth Part of 3.1416, we muſt take 6 for the Diameter ſought. For if 3.1416 be multiplied by the Square of 6; *viz.* by 36, the Product will be 113.0976; and if .5236 be multiplied by the Cube of 6; *viz.* by 216, the Product is likewiſe 113.0976, the Solidity equal to the Superficies.

Therefore, 6 is the true Anſwer.

Queſt. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Becauſe the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be

$$8 : 18 :: 6 : 13.5.$$

So the Diameter ſought $13\frac{1}{2}$.

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it would be

$$18 : 8 :: 6 : 2\tfrac{2}{3}.$$

So then the Diameter will be $2\frac{2}{3}$.

The Reaſon of theſe Operations, both in this and the laſt Queſtion, is from Algebra.

Queſt. 33. There are three Grenado-ſhells, of ſuch Capacity, that the ſecond Shell will juſt lie in the Concavity of the firſt, and the third in the Concavity of the ſecond. The Solidity of the Metal of the firſt Shell is equal to its Concavity, and the Solidity of the Metal in the ſecond, to the Concavity, is as 7 to 5; and the Solidity of the third, or leaſt Shell's Metal, to its Concavity, is as 9 to 4. Now, ſuppoſing the Diameter of the firſt, or greateſt Shell, to be 16

 Inches,

Inches, and allowing every ſolid Inch of Iron to weigh 4 Ounces; I demand the Diameter of the two leſſer Shells; and the Thickneſs and Solidity of Metal of every Shell; and alſo the Weight of every Shell?

The Cube of 16 is 4096; then,

1 : .5236 : : 4096 : 2144.6656.

The half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as alſo the Concavity.

.5236 : 1 : : 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the ſecond Shell.

The Sum of 7 and 5 is 12; then,

12 : 5 : : 1072.3328 : 446.805.

This 446.805 is the ſolid Content of the Concavity of the ſecond.

.5236 : 1 : : 446.805 : 853.333.

The Cube Root of 853.333 is 9.485, the Diameter of the leaſt Shell.

The Sum of 9 and 4 is 13; then,

13 : 4 : : 446.805 : 137.47846.

This 137.47846 is the ſolid Content of the Concavity of the third.

.5236 : 1 : : 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the leaſt Shell's Concavity.

From	16	the Diameter of the greateſt.
Subtr.	12.699	the Diameter of the ſecond.
Rem.	3.301	

Half is = 1.65 the Thickneſs of Metal of the greateſt.

From

From 12.699 the Diameter of the ſecond.
Subt. 9 485 the Diameter of the leaſt.

Rem. 3 2.4

Half is = 1.607 the Thickneſs of Metal of the ſecond.

From 9.485 the Diameter of the leaſt,
Subtr. 6 403 the Diameter of the Concavity.

Rem. 3.082

Half is = 1.541 the Thickneſs of Metal of the leaſt.

The Metal of the greateſt is 1072.33 ſolid Inches; which divide by 4 (becauſe every ſolid Inch is a Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the ſecond is 625.52 ſolid Inches; which divided by 4, the Quotient is 156.38 Pounds, the Weight of the ſecond.

The Metal of the leaſt Shell is 390.32 ſolid Inches; which divided by 4, the Quotient is 77.33 Pounds, the Weight of the leaſt.

The Diameter of the	ſecond Shell	12.699	Inches.
	leaſt Shell	9.485	
The Thickneſs of the Metal of the	greateſt	1.65	Inches.
	ſecond	1.607	
	leaſt	1.541	
The Weight	greateſt	268.08	Pounds.
	ſecond	156.38	
	leaſt	77.33	

A SHORT

A SHORT APPENDIX.

§ I. *Of* Gauging.

I SHALL not here give the whole Art of Gauging (there being several Books of that Art already in Print, written by better Hands;) but shall only lay down some short practical Rules, by which any Artificer, or others, may find the Quantity of Liquor in any Vessel upon Occasion.

Problem I.

To find the several Multipliers, Divisors, and Gauge-points, belonging to the several Measures now used in England.

282)1.0000(.003546 Multiplier for Ale Gallons.
231)1.0000(.004329 Multiplier for Wine Gallons.
268.8)1.000(.0037202 Multiplier for Corn Gallons.
2150.42)1.000(.00046502 Multipl. for Corn Bushels.

So, if the solid Inches in any Vessel be multiplied by the said Multipliers, the Product will be Gallons in

in the respective Measures; or dividing by the Divisors 282.231, or 268.8, the Quotient will likewise be Gallons.

Note, That 282 solid Inches is a Gallon of Ale or Beer-measure; 231 solid Inches is a Gallon of Wine-measure; 268.8 solid Inches is a Gallon, and 2150.42 solid Inches is a Bushel of Corn-measure.

For circular Areas, the following Multipliers and Divisors are to be used.

282).785398(.002785 Multiplier for Ale Gallons.
231).785398(.0034 Multiplier for Wine Gallons.
.785398)282.(359 05 Divisor for Ale Gallons.
.785398)231.(294.12 Divisor for Wine Gallons.
.785398)2150.42(2738 Divisor for Corn Bushels.
The Square Root of the Divisor is the Gauge-point.

The Gauge-point for Squares in	Ale-measure, is	16.79
	Wine-measure, is	15.2
	Malt-bushel, is	46.37
The Gauge-point for circular Figures in	Ale-measure, is	18.95
	Wine-measure, is	17.15
	Malt-bushel, is	52.32

Problem II.

To find the Area in Ale or Wine Gallons, of any rectilineal plain Figure, whether Triangular, Quadrangular, or Multangular.

TO resolve this Problem, you must, by Chap. I. Part II. find the Area in Inches, and then bring it to Gallons, by dividing that Area in Inches by the proper Divisor; *viz.* by 282 for Ale, or by 231 for Wine; or else by Multiplication, by .003546 for Ale, or by .004329 for Wine; and the Quotient or Product will be the Area.

Example.

Example. Suppose a Back or Cooler in the Form of a Parallelogram, or long Square, 250 Inches in Length, and 84.5 Inches in Breadth; What is the Area in Ale or Wine Gallons?

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches, which divide by 282, and the Quotient is 74.9 Gallons of Ale; or multiplied by .003546, the Product is 74.90928 Gallons, nearly the same; and if 21125 be divided by 231, or multiplied by .004329, it will give 91.44 Gallons of Wine.

By Scale and Compasses.

Extend the Compasses from 282 to 250, that Extent will reach from 84.5 to 74.9. And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Note, The Areas of all Superficies are always to be understood to be 1 Inch deep; otherwise it could not be said, that the Area of such a Parallelogram, Circle, *&c.* is so many Gallons.

Having found the Area of a Back or Cooler, the next Thing will be to find out the true Dipping or Gauging-place in that Back, that so the true Quantity of Worts may be computed at any Depth; which may be thus done.

1. When the Bottom of the Back is covered all over (of any Depth) with Worts, or other Liquor, then dip it in eight or ten several Places (more or less, according to the Largeness of the Back,) as remote and equally distant from each other as you can well do, noting down the wet Inches and decimal Parts of every Dip.

2. Divide the Sum of all those Dips by the Number of Places you dipped in, and the Quotient will be the mean Wet of all those Dips.

3. Lastly,

3. Lastly, find out such a Place by the Side of the Back (if you can) that just wets the same with that mean Dip, and make a Notch or Mark there for the true and constant Dipping-place of that Back.

Then if any Quantity of Worts (which covers the whole Back) be dipped, or gauged at that Place, and the wet Inches so taken be multiplied into the Area of the Back in Gallons, the Product will shew how many Gallons of Worts are in the Back at that time, provided the Sides of the Back do stand at Right-angles with the Bottom.

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area of it in Ale or Wine Gallons.

IF the Square of the Diameter be multiplied by .002785 for Ale, or by .003399 for Wine; or if it be divided by 359.05 for Ale, or by 294.12 for Wine, the Products or Quotients will be the respective Ale or Wine Gallons.

Example. Suppose the Diameter of the Circle be 32.6 Inches; What will be the Area in Ale or Wine Gallons?

The Square of 32.6 is 1062.76.

Then 359.05)1062.76(2.9599 Area in Ale Gallons.
And 294.12)1062.76(3.6133 Area in Wine Gallons.
Or 1062.76 × .002785 = 2.9598 Ale Gallons.
And 1062.76 × .0034 = 3.6133 Wine Gallons.

By

By Scale and Compasses.

Extend the Compasses from 18.95 (the Gauge-point for Ale) to 32.6 (the Diameter) that Extent will reach from 1 to a 4th Number, and from that 4th to 2.9599 Gallons. Or, extend the Compasses from 1 to 32.6, that Extent, turned twice over from .002785, will at last fall upon 2.9599.

For Wine extend from 17.15 (the Gauge-point for Wine) that Extent, turned twice over from 1, will at last fall upon 3.6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .0034, being twice turned over, to 3.6133 Wine Gallons.

PROBLEM IV.

The Transverse (or longest Diameter) and the Conjugate (or shortest Diameter) of an Ellipsis (or Oval) being given, to find its Area in Ale or Wine Gallons.

IF the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiplied by .002785 for Ale, or divided by 294.12, or multiplied by .0034 for Wine, the Quotient or Product will be the Ale or Wine Gallons required.

Example. Suppose the longest Diameter be 81.4 Inches, and the shortest Diameter be 54.6 Inches; What will be the Area of that Oval?

Multiply 81.4 by 54.6, and the Product is 4444.44; then,

.359.05)4444.44(12.38 Area in Ale Gallons.
294.12)4444.44(15.11 Area in Wine Gallons.
Or 4444.44 × .002785 = 12.38 Ale Gallons.
And 4444.44 × .0034 = 15.11 Wine Gallons.

By Scale and Compasses.

First, find a mean Proportional between 81.4 and 54.6, by dividing the Distance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional (that is, the Diameter of a Circle equal to the Oval.) Then extend the Compasses from 18.95 (the Gauge-point for Ale) to 66.6, that Extent, turned twice over from 1, will at last fall upon 12.38, Ale Gallons: And extend from 17.15 (the Gauge-point for Wine) to 66.6; that Extent, turned twice over from 1, will reach at last to 15.11 Wine Gallons.

PROBLEM V.

To find the Content in Ale or Wine Gallons of any Prism, whatsoever Form its Base is of.

FIRST, find its solid Content in Inches (by Sect. I, II, III. of Chap. II. Part II.) then divide that Content in Inches by 282 for Ale, or by 231 for Wine; the respective Quotients will be the Content in Wine or Ale Gallons.

Otherwise, you may find the Content of a Prism by finding the Area of its Base in Gallons (by Problem II. of this Appendix) and multiply that Area by

by the Tun's Height, or Depth within, the Product will be its Content in Gallons.

Example. Suppose a Tun, whose Base is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale and Wine Gallons is required.

The Length, Breadth, and Depth, being multiplied continually, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale Gallons: And divided by 231, the Quotient is 331.84 Wine Gallons: And by dividing by 2150.4, such a Cistern will be found to hold 35.65 Bushels of Corn.

By Scale and Compasses.

Extend the Compasses from 282 to 36.5, the Breadth of the Base, that Extent will reach from 49.3, its Length, to 6.38 Ale Gallons, the Area of the Base; then extend from 1 to 42.6, the Depth, that Extent will reach from 6.38, the Area of the Base, to 271.8 Gallons the Content.

Problem VI.

To find the Content of a Tun, whose Bases are alike and parallel, but unequal, being the Frustum of a Pyramid.

FIND the Area of each Base, and a mean Proportional between them, and multiply the Sum of those three by one third Part of the Depth or Height, and the Product is the Content.

 Example.

Example. Suppose a Tun, whose Bases are Parallelograms; the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the lesser Base 80, and its Breadth 56; and the Depth of the Tun 42 Inches; the Content in Ale and Wine Gallons is required.

Multiply 100 by 70, the Product is 7000, the Area of the greater Base; and 80 multiplied by 56, the Product is 4480, the Area of the lesser Base; then multiply the two Areas into each other; and the Product is 3136000, whose Square Root is 5600, a geometrical mean Proportional.

The greater Area	7000	add.
The lesser Area	4480	
The mean Proportional	5600	
	17080	
A third of the Depth	14	
	68320	
	17080	

282)239120(847.94 A. g.

231)239120(1035.15 W. g.

PROBLEM

PROBLEM VII.

To find the Content of a Tun, whose Bases are parallel and circular, it being the Frustum of a Cone.

YOU may find the Content as in the last Problem, by multiplying the Sum of the Areas of the two Bases, and a mean Proportional, by one third Part of the Depth.

But it will be a shorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and lesser Diameters add one third Part of the Square of the Difference of the Diameters; that Sum is the Square of a mean Diameter, which, divided by 359.05 for Ale, or by 294.12 for Wine, gives the Area of a mean Circle in Ale or Wine Gallons, which, multiplied by the Depth, gives the Content.

Example. Suppose the greater Diameter 80 Inches, and the lesser Diameter 71 Inches, and the Depth 34 Inches, the Content in Ale or Wine Gallons is required.

Multiply 80 by 71, and the Product is 5680; to which add 27 (a third Part of the Square of the Difference of the Diameters) and the Sum is 5707, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Gallons the Area; which multiply by 34 (the Depth,) and the Product will be 540 43 Gallons, the Content.

By Scale and Compasses.

Add the two Diameters together, and take half the Sum, which is 75.5, which take for a mean Diameter (though it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great;) extend the Compasses from 18.95 (the Gauge-point for Ale) to 75.5, the mean Diameter; that Extent will reach from 34 (the Depth) to a fourth Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compasses from 17.15 (the Gauge-point for Wine) to 75.5, that Extent will reach from 34, twice turned over, to 659.7 Gallons of Wine.

The Method used by the Gaugers for all such Tuns, is to take the Diameter in the Middle of every 10 Inches; that is, at five Inches from the Bottom, and at 15, and at 25, &c.

Then they find the Area to every one of these Diameters, and enter them in their Books. Then, when they survey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every 10 Inches they take the respective Areas, and remove the separating Point one Place towards the Right-hand; and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and so add all the several Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppose the Diameter at 5 Inches from the Bottom 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area answering to 64 Inches is 11.4073 Gallons; and to 67 Inches, is 12.5019 Gallons; and the Area to 70 Inches, is 13.6565 Gallons; and to 73, is 14.8413 Gallons. Then, supposing the Depth of the Liquor in the said Tun be

found to be 3.6 Inches: Now, to cast up this Gauge, first, in the Area answering to 64 Inches, being multiplied by 10, that is by removing the separating Point a Place towards the Right-hand, it will be 114 073 Gallons; and the next will be 125.019; and the next 136.565 Gallons. Now these three will be the Content to 30 Inches deep. Then, to find the Content of the 3.6 Inches, multiply the next Area 14.8413 by 3.6, and the Product is 53.4268: Add all together, and the Sum is the whole Quantity of Liquor in the Tun.

The Content at 10 Inches deep	114.073
The Content at the next 10 Inches	125.019
The Content at the next 10 Inches	136.565
The Content of the 3.6 Inches	53.427
The whole Quantity of Liquor in the Tun	429.084

Problem VIII.

To find the Drip or Fall of a Tun.

SUppose the Tun last mentioned was so placed, that when the Bottom is but just covered on one Side, the Liquor is 4 Inches deep on the Side opposite; How much must be allowed for the Fall of this Tun? That is, How much Liquor is there in the Tun?

The Diameter in the Middle of 4 Inches from the Bottom, is 61.6 Inches; and the Area answering thereunto is 10.568; which multiplied by 2 (that is, half 4,) the Product is 21,136 Gallons; and so much Liquor will just cover the Bottom.

But,

But, ſuppoſe it was ſet ſo much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the oppoſite Side juſt cuts between the Bottoms and Staves; How much Liquor will there be in the Tun?

Square the Bottom Diameter, and multiply that Square by the Top Diameter, and divide the laſt Product by the Sum of the Diameters, and to the Quotient add the Square of the Bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine; multiply the Quotient by the Depth, the Product is the Content.

The Bottom Diameter of the fore-mentioned Tun is 61 Inches; and the Diameter, at 30 Inches from the Bottom, is 71.5 Inches; the Square of 61 is 3721; which multiplied by 71.5, the Product is 266051.5; this divided by 132.5, (the Sum of the Diameters) the Quotient is 2007.936: To which add 3721 (the Square of 61,) and the Sum will be 5728.936; this divided by 1077.15, the Quotient is 5.3186; which multiplied by 30, the Depth, the Product is 159.558, the Gallons of Liquor in the Tun.

When the Fruſtum of a Cone or Pyramid is cut, by a diagonal Plane, through the Extremities of the Diameters, as the Liquor in the Tun repreſents, ſuch Solid is called a Hoof. (Vide *Ward*'s *Young Mathematician's Guide*, Page 414.)

If it be the Hoof of a ſquare Fruſtum, inſtead of dividing by 1077.15, divide by 846 for Ale, or by 693 for Wine. All the reſt of the Work is the ſame.

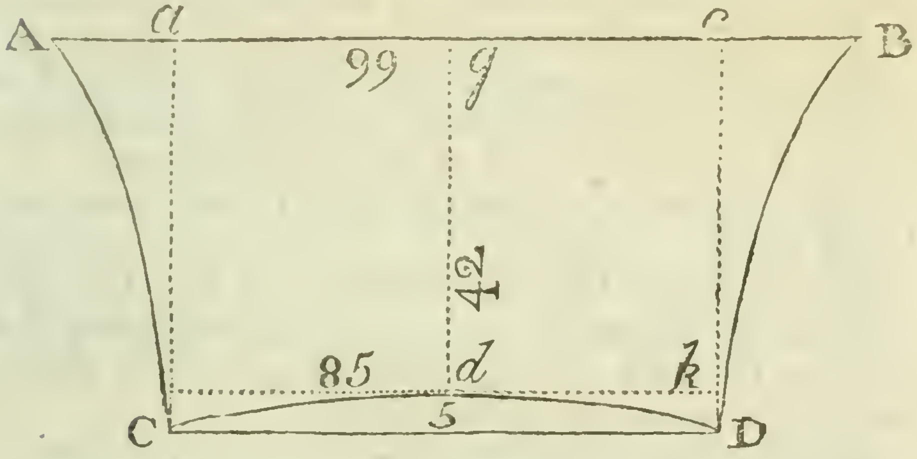
PROBLEM

Problem IX.

To gauge a Copper.

LET ABCD be a ſmall Copper to be gauged.

Take a ſmall Cord of Packthread, make one End faſt at A, and extend the other to the oppoſite Side of the Copper at B, where make it faſt, or cauſe ſome Perſon to hold it very ſtrait; then ſet one End of the Inſtrument in the Bottom of the Copper at C, and move it to and fro, till you find the neareſt Diſtance to the Thread (as at *a*:) This Diſtance, *a* C, is the Depth of the Copper, which ſuppoſe to be 47 Inches.



In like manner, ſet the End of the Rule upon the Top of the Crown at *d*, and take the neareſt Diſtance to the Thread, as *dg*, which ſuppoſe 42 Inches: This ſubtracted from *a* C, 47, the Remainder 5 is the Altitude of the Crown.

To find C D, the Diameter of the Bottom of the Crown.

Meaſure A B, the Diameter of the Top, which admit to be 99 Inches; then hold a Thread ſo as a Plummet at the End thereof may hang juſt over C, by

by which means you will find the Distance A *a*. Do the like on the other Side; so will you find also the Distance, B; which suppose 17.5 Inches each; add these two together, and subtract their Sum (*viz.* 35) from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown. The Diameter which touches the Top of the Crown, may be found by the Sliding-rule to be 65 Inches.

Now to find the Content of the Copper from the Crown upwards, that is, the Part A B *k b*, the Depth *g d* being 42 Inches, you may take the Diameter in the Middle of every 6 Inches of the Depth, which suppose to be as in the second Column of the following Table, the Numbers in the third Column are the respective Areas in Ale Gallons, found by Problem III. the fourth Column shews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part, A B *k b*; that is, so much as it will hold after the Crown is covered.

Now, if the Crown be taken for the Frustum of a Sphere, the Content (by the latter Part of Sect. II. Page 190,) will be found to be 28.75 Gallons.

But may be more readily found, very near the Truth, thus:

The Diameter C D was found to be 64, and the Area to this Diameter is 11.408; this multiplied by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Gallons, the Content of the Crown.

The Content of the Part *b k* D C is 57.935 Gallons; from which subtract the Content of the Crown, 28.52, and the Remainder is 29.415 Gallons, and so much Liquor will just cover the Crown.

Parts

Parts of the Depth	Diameter.	Areas.	Content of every 6 Inches.
6	95.3	25.2945	151.767
6	90.1	22.6095	135 657
6	85 0	20.1223	120.734
6	80	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66	12 1310	72.701
The Sum —		—	705.451
To just cover the Crown		—	29.415
The whole Content		—	794.866

By Scale and Compasses.

You may find the Areas answering to every one of the Diameters, thus:

Extend the Compasses from the Gauge-point to the Diameter; that Extent being turned twice over from 1, will at last fall upon the Area of that Circle: Or being turned twice over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compasses from 18 95 (the Gauge-point) to 95.3; that Extent, turned twice over from 6, will at last fall upon 151.76 Gallons, the Content of the first 6 Inches. And so of the rest.

PROBLEM

Problem X.

To compute the Content of any close Cask.

IN order to perform this difficult Part of Gauging, the three following Dimensions of the Cask must be truly taken;

Viz.	The Bung-diameter, The Head-diameter, The Length of the Cask,	within the Cask.

In taking these Dimensions, it must be carefully observed,

1. That the Bung-hole be in the Middle of the Cask; also, that the Bung-staff, and the Staff opposite to the Bung-hole, are both regular and even within.

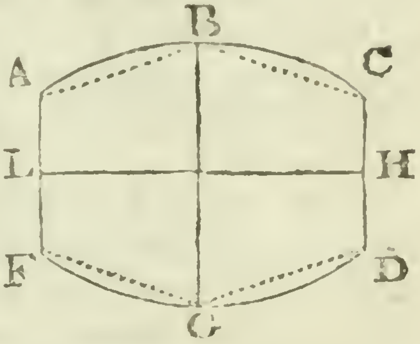
2. That the Heads of the Cask are equal, and truly circular; if so, the Distance between the Inside of the Chine to the Outside of its opposite Staff, will be the Head-diameter within the Cask, very near.

3. With a sliding Pair of Calipers (made for that Use) take the shortest Distance, or Length, between the Outsides of the two Heads; from that Length subtract $1\frac{1}{5}$ Inch (more or less according to the Largeness of the Cask) for the Thickness of the Head: The Remainder will be the Length of the Cask within. But if the Cask be empty, you may take the Length, by putting a strait Rod in at the Tap-hole, and allow for the Thickness of the Head.

Now by these Dimensions, some would think the Content of the Cask was perfectly limited; but it will be easy to perceive, by the following Figure, that the

the Diameters and Length of one Cask may be equal to those of another, and yet one of those Casks may contain several Gallons more than the other.

As for Instance, the Figure ABCDF is supposed to represent a Cask: Then it is plain, that if the outward curve Lines, ABC, and FGD, are the Bounds or Staves of the Cask, it must needs hold more than if the inner pricked Lines were the Bounds, or Staves; and yet the Bung-diameter BG, and Head-diameters CD and AF, and the Length LH, are the same in both those Casks.



Whence it appears, that no one general Rule can be given, by which the Content of all Sorts of Casks can be gauged: And therefore Gaugers do usually suppose every Cask to be in some of these Forms:

1. The middle Frustum of a Spheroid.
2. The middle Frustum of a parabolic Spindle.
3. The lower Frustums of two equal parabolic Conoids.
4. The lower Frustums of two equal Cones.

1. If the Staves of a Cask be very much curved (as the outward Lines of the last Figure;) then the Cask is supposed to be the middle Frustum of a Spheroid.

2. If the Staves (between the Bung and Head) be something less curved, then the Cask is taken to be the middle Frustum of a parabolic Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Cask is taken to be the lower Frustums of two equal parabolic Conoids,

abutting or joining together upon one common Base.

4. If the Staves between the Bung and Head be strait (as the pricked Lines in the last Figure,) then the Cask is taken to be the lower Frustums of two equal Cones, abutting or joining together upon one common Base.

There are several Rules laid down in Books of Gauging, for finding the Content of each Form; but I think the shortest and most practical Way is, to find such a mean Diameter, as will reduce the proposed Cask to a Cylinder: Thus,

Multiply the Difference of the Bung and Head Diameters by .7 for the Spheroid; by .65 for the second Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head-diameter, and the Sum is the mean Diameter.

Example. Suppose the Bung-diameter be 32 Inches, the Head-diameter 24 Inches, and the Length 40 Inches; the Content in each Variety is required.

The Difference between the Bung and Head-diameter is 8; which multiplied by .7, the Product is 5.6; which added to the Head-diameter, the Sum is 29.6, the mean Diameter: The Area answering to it will be found (by Prob. III.) to be 2.44 Ale Gallons; which multiplied by the Length, the Product is 97.4 Gallons; and so much is the Content, if it be the first Form.

Again; if the Difference of the Diameters 8 be multiplied by .65; the Product will be 5.2; which added to the Head-diameter, the Sum is 29.2, for the mean Diameter; and the Area answering to it is 2.3746 Gallons; which multiplied by 40 (the Length) the Product is 94.98 Gallons, the Content, if it be of the second Form.

Again;

Again; if the Difference 8 be multiplied by .6, the Product is 4.8; which added to the Head-diameter, the Sum is 28.8, the mean Diameter: The Area anſwering to it is 2.31 Gallons; which, multiplied by 40, gives the Content 92.4 Gallons, for the third Form.

Again; the Difference 8, multiplied by .55, the Product is 4.4; which added to the Head-diameter, makes the mean Diameter 28.4; the Area anſwering to it is 2.2463; which multiplied by 40, the Product is 89.85 Gallons, for the fourth Form.

By Scale and Compaſſes.

Extend the Compaſſes from the Gauge-point 18.95 to the firſt mean Diameter 29.6; that Extent will reach from the Length 40 to a fourth Number, and then to the Content, 97.4 Gallons.

Again; extend from 18.95 to 29.2 (the ſecond mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 92.98 Gallons.

Again; extend from 18.95 to 28.8 (the third mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 92.4 Gallons.

Again; extend from 18.95 to 28.4 (the fourth mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 89.85 Gallons.

Although I have all along made uſe of the Line of Numbers upon the common Two-foot or Eighteen-inch Rules, for the Reaſon mentioned in the *Preface*; yet the Rules may eaſily be applied to the Sliding-rule, thus: To find the Area of a Circle in Gallons, ſet the Gauge-point upon D (that is, a ſingle Line of Numbers) (to 1 upon C, that is, a double Line;)

 then

then againſt any Diameter upon D, is the Area upon C, thus;

To find the Content of the Caſk, laſt mentioned, the firſt Form.

Set the Gauge-point 18.95 upon D, to the Length 40 upon C; then (againſt the mean Diameter) 29.6 upon D, is 97.4 Gallons, the Content upon C.

And againſt 29.2 (the next mean Diameter) on D, is 94.98 Gallons on C.

And againſt 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And againſt 28.4 (the laſt mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE

A TABLE *of the Segment of a Circle, whose Area is Unity.*

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
1	.0017	99	.9983	26	.2066	74	7924
2	.0048	98	.9952	27	.2178	73	.7832
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7360
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	.3241	64	.6759
12	.0680	83	.9320	37	.3364	63	6636
13	.0764	87	.9236	38	.3487	62	.6513
14	.0851	86	.9149	39	.3611	61	6389
15	.0941	85	.9059	40	.3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	1224	82	.8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	.5762
20	.1424	80	.8576	45	.4364	55	5636
21	.1526	79	8474	46	.4491	54	.5509
22	.1631	78	.8369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	.5250
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Use of the Table of Segments

Is to find the Ullage, or Quantity of Liquor remaining in a Cask, whose Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask.

The RULE *is*;

To the wet or dry Inches of the Bung-diameter, add a competent Number of Cyphers; then divide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiplied by the whole Content of the Cask, the Product shews the Quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Cask in Form of a Cylinder, whose Bung-diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; How many Gallons are wanting to fill the Cask?

Divide the dry Inches 13, by 29 the Bung-diameter, and the Quotient is 448; find the two first Figures .44 under V. S. and the Segment against it is .4238; to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiplied by the Content of the Cask, the Product will be 34.664 Gallons; and so much the Cask wants of being full.

Note, If the Cask be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a spheroidal Cask, then use the following Method.

1. By

1. By the Bung and Head-diameter, find ſuch a mean Diameter as, you judge, will reduce the propoſed Caſk to a Cylinder, and then find its Content.

2. From the Bung diameter ſubtract the mean Diameter, and take half the Difference.

3. From the wet Inches ſubtract the ſaid Half-difference; reſerve this Difference, then uſe the Proportion:

As the mean Diameter is to 100
(the Diameter of the tabular Circle,)
So is the reſerv'd Difference
to a verſed Sine in the Table.

Then, if the tabular Segment be multiplied into the Content (as before) the Product will be the Quantity of Liquor in the Caſk.

Example. Let the Caſk be the ſame as in Page 325, of the firſt Form, where the Bung-diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and ſuppoſe the wet Inches 19, to find the Quantity of Liquor in the Caſk.

From	32	From	19	
Subtr.	29.6	Subtr.	1.2	
Rem.	2.4	Rem.	17.8	reſerved.
Half	1.2			

29.6 : 100 : : 17.8 : .60, the V. S.

The Segment to 60 is .6265, which multiplied by 97 4, the Content, the Product is 61 Gallons, the Quantity of Liquor in the Caſk.

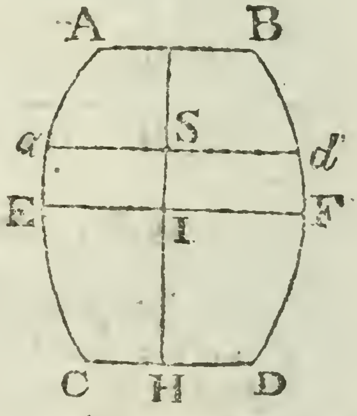
If the dry Inches have been given, by the same Method, you might have found the Ullage, or what the Cask wanted of being full.

To find what Quantity of Liquor is in a Cask, when its Axis is perpendicular to the Horizon; *viz.* when it stands upright upon one of its Heads.

To do this, you must know how to calculate the Area of any Circle, between the Bung and Head, whose Distance from the Bung, or Middle of the Cask, is given; which may be done by this Proportion.

As the Square of half the Length of the Cask is to the Difference between the Bung and Head-areas; so is the Square of any Circle's Distance from the Bung, to the Difference between the Bung-area and the Area of that Circle; *viz.* the Area of the Liquor's Surface.

Then, from the Bung-area, subtract one-third Part of the aforesaid Difference; *viz.* between the Bung-area and the Area of the Liquor's Surface: Multiply the Remainder by the Liquor's Distance from the Bung, and the Product will shew what Quantity of Liquor is either above or under half the Content of the Cask.



Example. Let us again suppose the Cask, in Page 326, whose Length is 40 Inches, Bung-diameter 32, and Head-diameter 24, and suppose the wet Inches, SH, 26 Inches.

The Square of half the Length is 400, the Distance of the Liquor's Surface from the Bung SI is 6, the Square of which is 36; the Area of the Bung D .2.8519 Ale Gallons, and the

the Area of the Head D. 1.6042; the Difference 1.2477. Then,

400 : 1.2477 : : 36 : .11229

One-third is = .0374

From	2.8519	Bung-area,
Subtr.	.0374	a Third of the Difference.
Rem.	2.8145	
	6	mult. Distance from the Bung.
	16.8870	Content above the Bung.
Add	48.7	half the Content of the Cask.
	65.5870	the Quantity of Liquor in the Cask.

PROBLEM IX.

Gauging of MALT.

TO find the Quantity of Malt in a Cistern, or upon a Floor.

First, Find the Area of the Base in Bushels, by multiplying the Length by the Breadth, and dividing the Product by 2150.42, or only by 2150; and multiply that Area by the mean Depth (How to take the mean Depth, see Problem II.) If the Base be circular or oval, divide by 2738, (see Problem I.)

Example.

Example. There is a Ciſtern, whoſe Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; What is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, and the Quotient is 2.1097 Buſhels, the Area of the Bottom at 1 Inch deep; which multiplied by the Depth 43.6, the Product is 91.98 Buſhels, the Content.

Example. Suppoſe a Quantity of Malt upon a Floor, whoſe Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches; How many Buſhels are there?

Multiply 245 by 184, and the Product is 45080; which divided by 2150, the Quotient is 20.967, the Area of the Baſe; which multiplied by the mean Depth, the Product is 117.4 Buſhels, the Content.

By the Sliding Rule.

There is an inverted Line of Numbers upon ſome Sliding-Rules marked with the Letter M, which was contrived purpoſely for Gauging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all of theſe are of equal Radius, and all work together: Thus ſet the Length and Breadth againſt one another upon the inverted Line, and that which ſlides by it; then, on the other Edge of the Rule againſt the Depth, you will find the Content in Buſhels. Thus, in the firſt Example, ſet 54 upon the Slider againſt 84 upon the inverted Line; and then, againſt 43.6 upon the other Part of the Rule, is 91.98 upon the Slider.

Again; in the ſecond Example, ſet 184 upon the Slider to 245 upon the inverted Line; and againſt 5.6 upon the other Part of the Rule, is 117.4 upon the Slider.

§ II. *Of* Land Measuring.

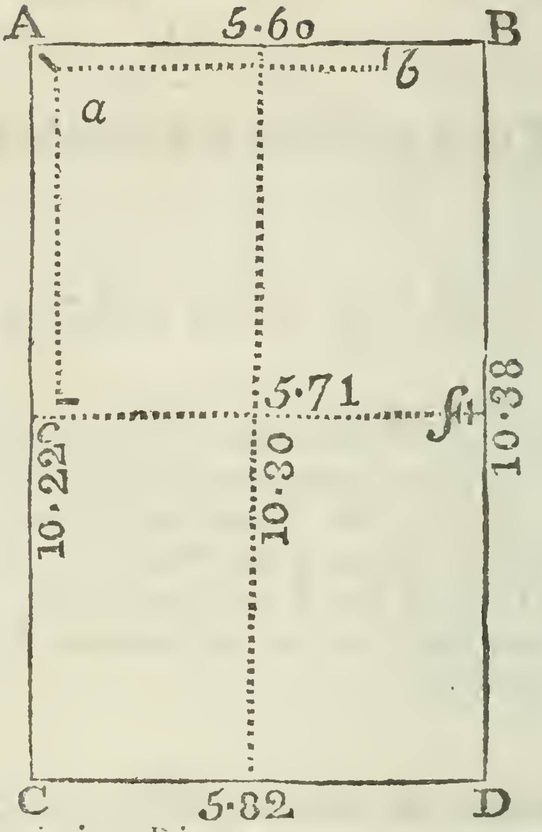
I SHALL not here give the whole Art of *Surveying*, but such practical Rules only as may be useful to the Country Grasiers and Farmers, and by which Means they may find the true Content of any Piece of Land, and that by the Chain only; or, for want of that, with a Pole or Stick of half a Rod in Length.

Problem I.

To find the Content of a Piece of Land in the Form of a right-angled Parallelogram, or long Square, or which is something near that Form.

TO know whether any Angle in the Field be a Right-angle, or not, you may take a Piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or square; and, with a Saw, cut two Kerfs, crossing each other at Right-angles; and bore a Hole in the Middle of the Backside, to put it upon the End of a Stick. This will represent the Instrument called a Cross.

Suppose



Suppose you would observe the Angle A, to know whether it be a Right-angle (or nearly so;) prick up your Stick, with the Cross upon it, at a little Distance from the Fence; as at *a*; and having set up two Marks, as at *b* and *c*, of equal Distance from the Fence, turn one of the Slits directly towards *b*; and then if the other be directly pointing to *c*, it is a Right-angle.

To measure such a Piece of Ground as this Figure above: If you measure round, and add the opposite Sides together, and take half the Sum (if they be not equal;) or else measure down about the Middle of the Length, and Middle of the Breadth; thus, the Side A B being measured, it will be 5 60 (that is, 5 Chains and 60 Links;) and the opposite Side C D is 5 Chains 82 Links; the half Sum of them is 5.71: And the Side B D is 10.38; and the Side A C 10.22; and the half Sum of them is 10.30 (it will be the same Thing, if you measure about the Middle of the Length and Middle of the Breadth;) then multiply this mean Length and mean Breadth together; *viz.* 10.30, by 5.71, and the Product is 58.8130; which divide by 10 (because 10 Square Chains is an Acre) by removing the separating Point one Place towards the Left-hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 5 Places, and it will be 3.52520; which 3 towards the

the Left-hand are 3 Roods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left-hand are 21 Perches.

	A.	R.	P.
So the whole Content is ———	5	3	21

See the Work.

```
   5.71
  10.30
 ------
  17130
  5710
 -------
 5.88130
       4
 -------
 3.52520
      40
 --------
21.00800
```

A.	R.	P.
5	3	21

Note, The Chain here made use of, is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But, because every Man that may have Occasion to measure a Piece of Land, cannot procure a Chain, I will therefore shew how you may measure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches; which Stick divide into five equal Parts, so will the whole Rod be divided into ten Parts, and will thereby be adapted to Decimal Arithmetic.

But, because each of those Parts of the Stick are something large (each Part being 19 Inches and 8 Tenths) it will be necessary to take your Dimensions to half of one of those Parts; and then, for that half Part, set 5 in the Place of Seconds, thus, suppose 3 Parts and a half, set it down thus, .35.

Problem II.

LET us suppose a Field in the Form of a long Square, whose Length is 45 Rods 5 Parts and a half, and the Breadth 31 Rods 4 Parts and a half; What is the Content?

Multiply the Length and Breadth together, and divide the Product by 160 (because 160 Square Rods are an Acre) and the Quotient is Acres.

```
    45.55
    31.45
  -------
    22775
   18220
   4555
 13665
 --------
 1432.5475
```

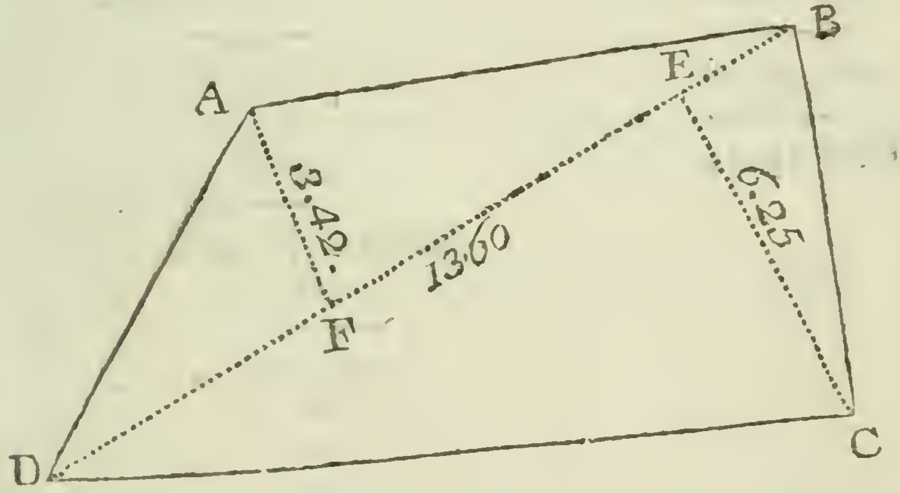
16|0)143|2(8 A. R. P.
128 *Facit* 8 3 32

4|0)15|2(3
12

32

PROBLEM III.

SUppose a Piece of Ground in the Form of a Trapezium; the Diagonal BD 13 Chains 60 Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular AF 3 Chains 42 Links; What is the Content?



Multiply the Diagonal by half the Sum of the Perpendiculars. See Sect. VI. of Chap. I. Part II.

CE=6.25	13.60=BD	
AF=3.42	4.83	
Sum 9.67	4080	
	10880	A. R. P.
Half 4.83	5440	*Facit* 6 2 11
	6.56880	
	4	
	2.27520	
	40	
	11.00800	

By Rods, thus:

CE=25 Rods.	19.34	
AF=13.68	54.4=BD	
Sum 38.68	7736	
	7736	
Half 19.34	9670	
	16|0)105|2.096(6	
	96	
	4|0)9|2(2	
	8	
	12	
		A. R. P.
		Facit 6 2 12

To

To take the Dimensions of the Field.

Begin at the Angle B, and measure in a direct Line towards D; but when you come at E, set up your Cross, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then are you just in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards and forwards till it does so; then measure the Perpendicular, and set down the Chains and Links, or the Rods and Parts; then continue your Measure towards D; but when you come to F, set up your Cross, and try (as is above directed,) whether you be in the Place where the Perpendicular will fall. Then measure the Perpendicular A F, and set down the Chain and Links, or Rods and Parts; then continue your Measure to D, and set down the Measure of the whole Diagonal. This Way of measuring is very exact and true; but the common Way used by the Graziers and Farmers, is to measure round the Field, and to take half the Sum of the opposite Sides for a mean Side; but the last mentioned Piece of Ground, being measured so, will come to

A. R. P. R. P.
7 0 22, which is 2 10 more than the Truth.

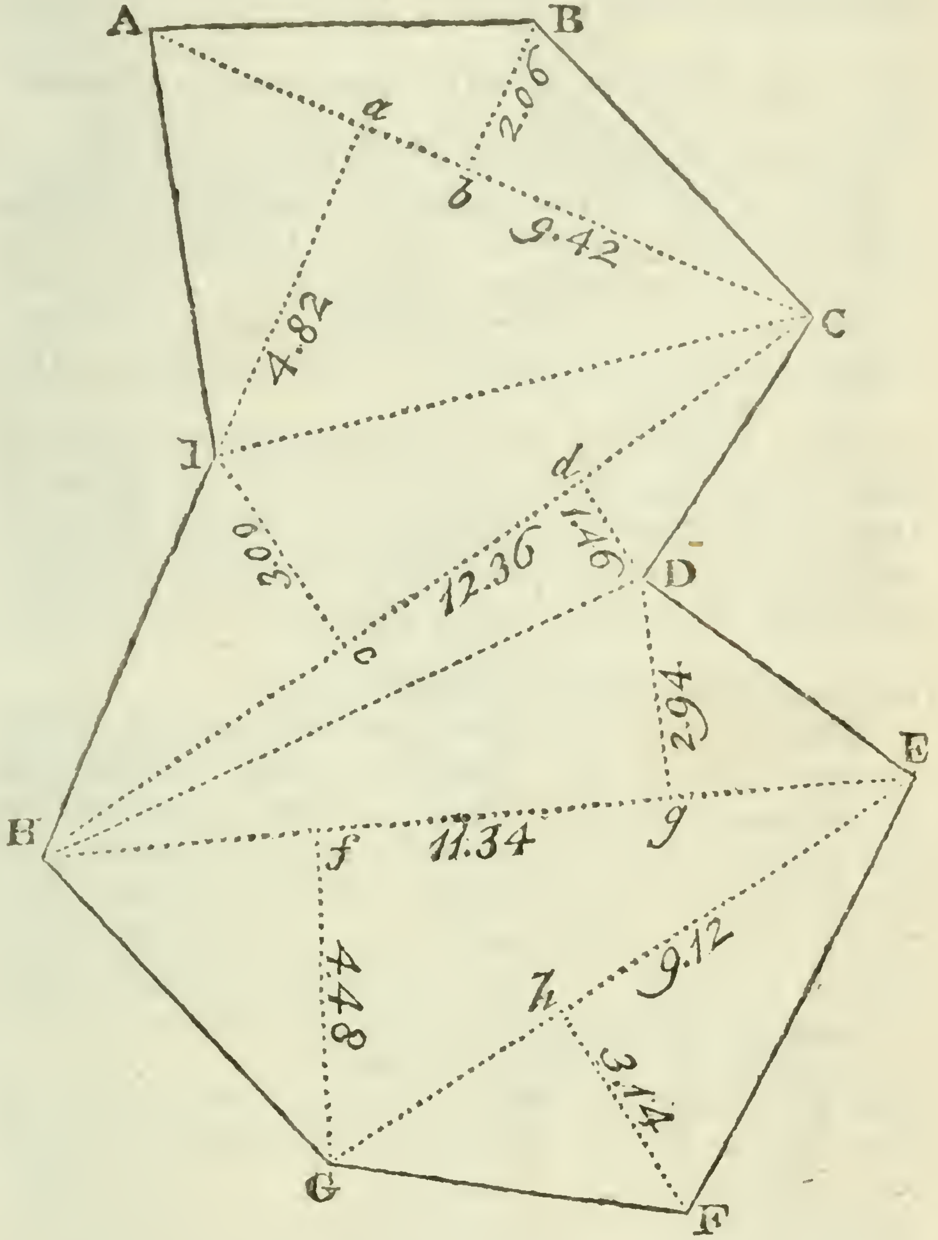
Problem IV.

How to meaſure an irregular Field.

THE Way to meaſure irregular Land, is to divide it into Trapeziums and Triangles, thus:

Firſt, View over the Field, and ſet up Marks at every Angle, and by thoſe Marks you may ſee where to have a Trapezium, as A B C I in the following Figure.

Then



Then begin and measure in a direct Line from A towards C; but when you come to (a), set up your Cross, and try whether you be in a Square to I (as is before directed;) and then measure the Perpendicular a I, which is 4.82; then measure forward again towards C, but when you come to (b) set up your

Croſs, and try whether you be in the Place where the Perpendicular will fall; then meaſure the Perpendi-
Ch. L.
cular b B, which is 2.06; then continue your Meaſure
Ch. L.
to C, and you will find the whole Diagonal 9.42.

Then proceed to meaſure the Trapezium CDHI, beginning at C, and meaſuring along the diagonal Line towards H; but when you come at (d), ſet up your Croſs, and try if you be in the Place where the Perpendicular will fall: Meaſure the Perpendicular d D,
Ch. L.
which is 1.46, and then meaſure forward till you come at (c,) and there, with your Croſs, try if you be right in the Place where the Perpendicular will fall, and meaſure the Perpendicular c I, which is 3 Chains; and from (c) continue your Meaſure to H, and you
Ch. L.
will find the whole Diagonal 12.36.

Then proceed to meaſure the Trapezium HGED, beginning at H, and meaſuring along the diagonal Line towards E; but when you come to (f) try with your Croſs, if you be in the Place where the Perpendicular will fall; and meaſure the Perpendicular f G, which is 4.48; then continue on your Meaſure from (f) till you come to (g), and there try if you be in a Square with the Perpendicular g D; and meaſure the
Ch. L.
ſaid Perpendicular, which is 2.94; then meaſure on from (g) to E, and you will find the whole Diagonal
Ch. L.
to be 11.34.

Then meaſure the Triangle EFG, beginning at E, and meaſuring along the Baſe EG, till you come at (h,) and there with your Croſs try if you be in the Place where the Perpendicular will fall; and mea-
Ch. L.
ſure the Perpendicular h F, which is 3.14, continue your Meaſure to G, and you will find the whole Baſe
Ch. L.
to be 9.12; ſo you have finiſhed your whole Field.

I have

I have been the larger upon the Explanation of the Problem, because most Grounds lie in such irregular Forms.

Cast up the three Trapeziums severally, and also the Triangle; and add all the several Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the Work.

bB=2.06	9.42	See Sect. VI. Chap. I. Part II.
aI=4.82	3.44	
Sum 6.88	3768	
	3768	
Half 3.44	2826	
	3.24048=Area of ABCI.	
dD=1.46	12.36	
cI=3.00	2.23	
Sum 4.46	3708	
	2472	
Half 2.23	2472	
	2.75628=Area of CIHD.	
fG=4.48	11.34	
gD=2.94	3.71	
Sum 7.42	1134	
	7938	
Half 3.71	3402	
	4.20714=Area of HGED.	

```
   Base=9.12              See Sect. V. Chap. I.
        ----                    Part II.
   Half=4.56
Perpend.  3.14
         -----
          1824
          456
         1368
        ------
       1.43184=Area of the Trian. EFG
       3.24048=Area of ABCI.
       2.75628=Area of CIHD.
       4.20714=Area of HGED.
       --------
Sum   11.63574=Area of the Whole.
             4
      --------
       2.54296
            40
       --------
       21.71840
       --------
```

See Sect. V. Chap. I. Part II.

	A.	R.	P.
Facit	11	2	21

FINIS.

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